Combined bispectrum-filtering techniques for radar output signal reconstruction in ATR applications

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ABSTRACT

In automatic target recognition applications, an important task is to obtain denoised signal signatures of the object. In this paper, the reconstruction of 1-D deterministic signals, for example, range profiles, corrupted by random signal shift and additive white Gaussian noise using 2-D bispectrum is considered. Combined bispectrum-filtering techniques based on smoothing the noisy bispectrum estimates by 2-D linear and nonlinear filters are proposed. It is shown that bispectrum estimates obtained by the conventional direct bispectrum estimator are corrupted by fluctuation errors and are biased. The performance of the proposed bispectrum-based signal reconstruction methods is analysed using two conventional criteria - the reconstructed signal fluctuation variance and bias. The numerical simulation results show that 2-D filtering of real and imaginary components of noisy bispectrum estimates is most efficient in the sense of minimum MS errors.

Keywords: smoothing of bispectrum estimate, bispectrum-filtering signal reconstruction techniques, linear and nonlinear scanning window filters

1. INTRODUCTION

Automatic target recognition (ATR) in radars and sonars is often performed by means of analysis of object signatures[1,2]. However, in practice target signatures, for example, range profiles[3,4], are distorted due to influence of noise, heterogeneity of the propagation medium, vibrations of the radar platform and antenna. The attempts to better extract the target signatures by processing (statistical averaging) of several signal realizations run into necessity to also take into account the target motion[4], and this makes the data processing algorithms rather complicated. Moreover, it is often quite difficult to design such algorithms.

In this sense, bispectral methods of signal processing seem to be a useful tool, especially taking into account that these techniques are able to retain useful phase information[5,6]. Data processing, analysis and extracting useful information by signal bispectrum estimates have been widely applied in various fields, e.g., in astronomy[5-7], medicine[8], underwater acoustics[9], non-destructive evaluation[10], etc. Radar applications of bispectral methods have been also considered[11-13].

The majority of investigations dealing with bispectrum analysis are dedicated to the problems of detection of signal non-Gaussianity by bispectrum estimates in aforementioned applications. And relatively small part of papers[5,6,8] focuses on more complicated and poorly studied problem, namely, signal (image) reconstruction by bispectrum estimates. In these applications, optimal linear filtering or prewhitening techniques often become non-effective tools of noise suppression since they need well-defined a priori knowledge about signal and noise characteristics. In other words, the performance of the reconstruction system degrades if less a priori knowledge about signal and noise characteristics is at hand.

Even though the methods reconstructing a signal from bispectrum data are suboptimal, they have several principal advantages that distinguish them from conventional correlation and spectral analysis[14]. These advantages are the following:

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- bispectrum-based methods require minimum a priori knowledge about signal and noise characteristics, e.g., the task of reconstruction of unknown signal waveform corrupted by noise with unknown variance can be solved in contrast to Wiener filtering that can not be applied for the considered case;
- bispectrum signal reconstruction technique performance does not sufficiently worsen with less a priori knowledge about signal and noise characteristics, in particular, if there is a limited information about object motion;
- recovery of the complex Fourier spectrum of the signal using bispectrum data is possible, hence, the important signal phase information is not lost as this is the case for power spectrum;
- bispectrum estimates are non-sensitive to random and deterministic temporal signal shifts, this provides an opportunity to design robust non-Gaussian signal reconstruction algorithms in Gaussian noise environment; the presence of such random shifts as well as the short pulse-like appearance of output signals typical for radars severely restrict the application of scanning window filters to direct denoising of observed signal realizations.

Robustness of the signal bispectrum reconstruction techniques to interference depends, first of all, upon bispectrum estimate accuracy. Thus it depends on input signal-to-noise ratio (SNR) and the number of observed realizations that participate in ensemble averaging of bispectrum estimates[15,16]. In radar and sonar applications the most important task is to recover data in case of "weak" signals, i.e. if the input SNR is low. Below we concentrate just on this case.

There are two general approaches to spectral estimation enhancement (that can be also applied in bispectral analysis): estimate smoothing and/or averaging[14]. Ensemble averaging, of course, improves bispectral estimates. However, for low input SNR and a limited number of processed observations the obtained bispectral estimates still need improvement in order to provide appropriate quality of reconstructed signals and desirable accuracy of their parameter evaluation. Smoothing of bispectrum estimates can additionally reduce reconstructed signal fluctuation variance, but it also introduces distortions, and, therefore, increases reconstructed signal bias[15]. Hence, one has to get some tradeoff between reconstruction signal fluctuation variance and bias.

Searching for improving the performance of bispectrum signal reconstruction system in the sense of obtaining reliable compromise between aforementioned variance and bias run us into idea of smoothing the noisy 1-D signal Fourier spectra recovered from noisy bispectrum estimates by 1-D linear and non-linear filters[17]. This approach presumed the smoothing of 1-D amplitude and phase signal Fourier spectra recovered by indirect bispectrum estimate. This method produced some improvement in comparison to conventional bispectrum techniques[5, 6]. However, the combined "bispectrum-filtering" technique proposed in[17] has some shortcomings dealing with rather considerable fluctuation errors that are always present in 2-D noisy bispectrum estimates and leak into 1-D signal Fourier spectra recovered from bispectrum estimates.

In order to partly alleviate these drawbacks, the application of smoothing 2-D bispectrum estimates that have been obtained by indirect bispectrum estimate technique[16] by 2-D filters with different scanning window sizes has been proposed in[15]. This paper is an attempt to extend and investigate the combined "bispectrum-filtering" techniques for direct bispectrum estimate methods[16]. In contrast to indirect bispectrum estimate technique, the direct bispectrum estimate technique requires less computation time. This is because the latter one employs FFT algorithm. Besides, direct method is more flexible since there are several different ways of bispectrum data smoothing. In other words, several different "bispectrum-filtering" techniques can be developed.

The paper organization is the following. In Section 2, the main peculiarities of direct bispectrum estimates and signal reconstruction from bispectrum data are described briefly. The novel "bispectrum-filtering" techniques based on signal Fourier spectrum recovery from noisy bispectrum estimates smoothed by linear and nonlinear 2-D filters are proposed in Section 3. The numerical simulation results and the comparative analysis data for the conventional bispectrum reconstruction technique and the proposed "bispectrum-filtering" techniques are presented in Section 4. Finally, the conclusions and recommendations are given in the Section 5.

2. PROBLEM STATEMENT

2. 1. Mathematical Background

Suppose that a real valued stationary 1-D time stochastic process $\{x^{(m)}(i)\}$ $(i=0,1,2,\dots,I-1)$ is observed at the digital signal reconstruction system input where its m-th realization ($m=1,2,\dots,M$) can be presented as

$$x^{(m)}(i) = s(i - \tau^{(m)}) + n_G{}^{(m)}(i), \tag{1}$$

where $n_G^{(m)}(i)$ is the m-th realization of additive noise supposed to be white Gaussian with zero mean and sample variance $\sigma_G^{(m)2}$; $\tau^{(m)}$ is a random shift of the original deterministic signal $s(i)$, for which its triple autocorrelation is assumed to be non-zero. White Gaussian noise (WGN) $n_G^{(m)}(i)$ is supposed uncorrelated with the original signal $s(i)$ in (1).

Let us consider the problem of reconstructing the unknown waveform. The goal is to obtain with bispectrum method an estimate with minimal error in some predefined sense. The assumptions are:
- some parameters of backscattered signal are supposed to be a priori unknown;
- the original signal triple autocorrelation function and bispectrum (Fourier transform of triple autocorrelation function) are to be non-zero;
- the original signal is supposed real-valued and its Fourier spectrum is Hermitian function;
- interference is additive WGN with zero mean and random shift of original signal.

While solving this signal waveform reconstruction problem we choose the commonly used criteria: fluctuation variance (MSE) and bias (systematic error), it is desirable to decrease both. If a human is to analyze the reconstructed signal and recognize the target(s), a visual fidelity criterion must be used in performance evaluation for displayed signals. Both quantitative and visual fidelity criterions are important for predicting the performance of some ATR methods that can be applied to processing of the reconstructed signals.

In order to recover the original signal from noisy observations (1) by traditional bispectrum signal reconstruction technique[6] the following steps should be performed: 1) estimate signal bispectrum; 2) recover signal magnitude and phase Fourier spectra from bispectrum estimate, and 3) carry out inverse Fourier transform of the recovered complex Fourier spectrum.

Complex valued 2-D bispectrum estimate of the observations (1) obtained by direct method[16] is be defined as

$$\begin{aligned} \hat{\dot{B}}_x(p,q) = \left\langle \hat{\dot{B}}_x{}^{(m)}(p,q) \right\rangle_M = \\ \left\langle \dot{X}^{(m)}(p)\dot{X}^{(m)}(q)\dot{X}^{(m)*}(p+q) \right\rangle_M =, \\ \left| \hat{\dot{B}}_x(p,q) \right| \exp[j\hat{\gamma}_x(p,q)] \end{aligned} \tag{2}$$

where $\hat{\dot{B}}_x^{(m)}(p,q)$ is the m-th bispectrum estimate realization; $\dot{X}^{(m)}(...) = DFT[x^{(m)}(i)]$ is the 1-D direct Fourier transform (DFT) of (1); $\left|\hat{\dot{B}}_X(p,q)\right|$ and $\hat{\gamma}_X(p,q)$ are the magnitude and phase bispectrum estimates, respectively; $p=-I/2+1,...,I/2-1,$ and $q=-I/2+1,...,I/2-1$ are the sample indices of independent frequencies in bispectrum domain; $<\ldots>_M$ denotes the ensemble averaging the observed (processed) M realizations; asterisk denotes complex conjugation and $j = \sqrt{-1}$.

For the signal and additive noise model (1), the noisy bispectrum estimate (2) can be rewritten as

$$\hat{\dot{B}}_x(p,q) = \dot{B}_s(p,q) + \dot{N}(p,q), \tag{3}$$

where $\dot{B}_S(p,q)$ is the original signal bispectrum, and $\dot{N}(p,q)$ is the noise induced component that leaks into bispectrum domain from system input. The latter term can be expressed as

$$
\begin{aligned}
\dot N(p,q) = {} & \dot S(p)\dot S(q)\left\langle \dot N^{(m)*}(p+q)e^{-j\tau^{(m)}p}\right\rangle_M + \dot S(p)\dot S^*(p+q)\left\langle \dot N^{(m)}(q)e^{j\tau^{(m)}q}\right\rangle_M + \\
& + \dot S(q)\dot S^*(p+q)\left\langle \dot N^{(m)}(p)e^{j\tau^{(m)}p}\right\rangle_M + \dot S(p)\left\langle \dot N^{(m)}(q)\dot N^{(m)*}(p+q)e^{-j\tau^{(m)}p}\right\rangle_M + \\
& + \dot S(q)\left\langle \dot N^{(m)}(p)\dot N^{(m)*}(p+q)e^{-j\tau^{(m)}q}\right\rangle_M + \dot S^*(p+q)\left\langle \dot N^{(m)}(p)\dot N^{(m)}(q)e^{j\tau^{(m)}(p+q)}\right\rangle_M + \\
& \left\langle \dot N^{(m)}(p)\dot N^{(m)}(q)\dot N^{(m)*}(p+q)\right\rangle_M
\end{aligned}
\quad . \qquad (4)
$$

From formula (4) we see that:

a) for increasing number of realizations M, the term $\left\langle \dot N^{(m)}(p)\dot N^{(m)}(q)\dot N^{(m)*}(p+q)\right\rangle_M$ tends to 0 if WGN in (1) is supposed zero mean; the real and imaginary components of this complex valued random process are not Gaussian (see the histogram in Fig. 1,a) although for large M it approximates Guassian pdf according to the central limit theorem (see the histogram in Fig. 1,b). These histograms have been obtained for SNR_{inp}=0, when only the last term in (4) is non-zero.

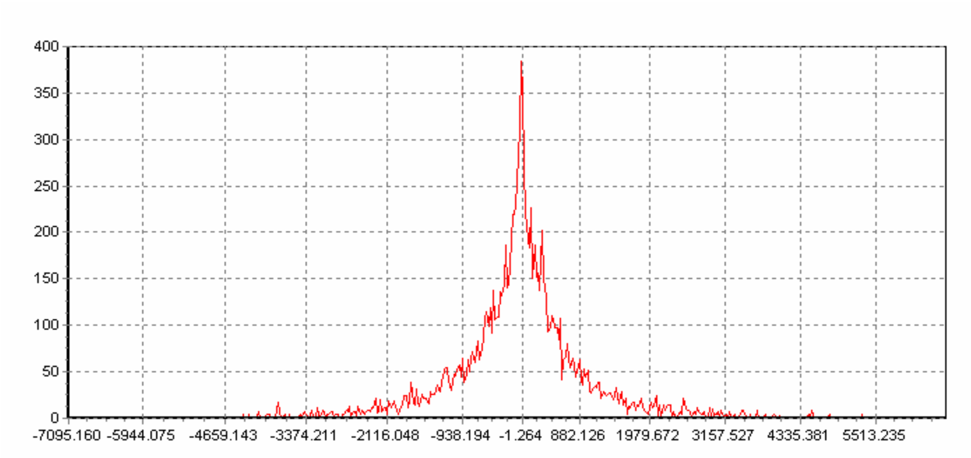

Fig. 1,a. Histogram of bispectrum real component values for signal-absence case, M=1

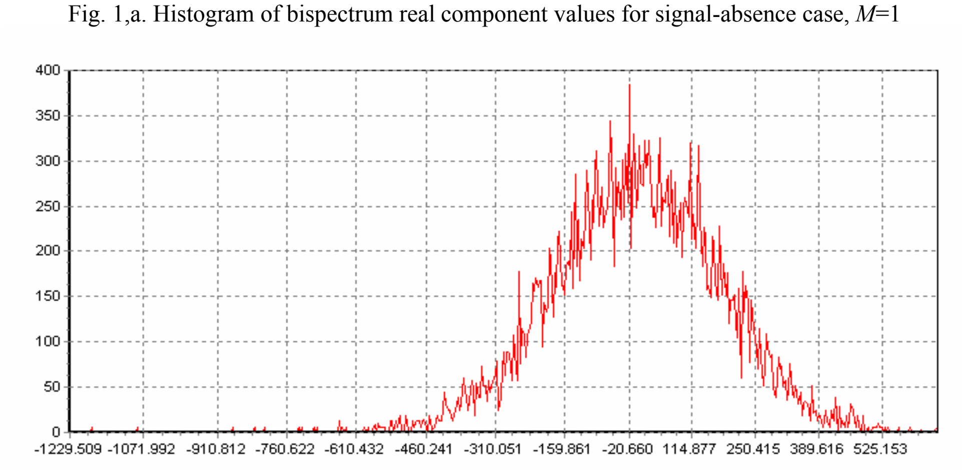

Fig. 1,b. Histogram of bispectrum real component values for signal-absence case, M=50

b) since we consider additive WGN with zero mean, hence, for large M also

$$\left\langle \dot N^{(m)*}(p+q)e^{-j\tau^{(m)}(p+q)}\right\rangle \cong \left\langle \dot N^{(m)}(q)e^{j\tau^{(m)}q}\right\rangle \cong \left\langle \dot N^{(m)}(p)e^{j\tau^{(m)}p}\right\rangle_M \cong 0 \ ;$$

c) the rest of noise induced terms in (4) are the "complex signal Fourier spectrum depending" and "random shift ($\tau^{(m)}$) depending". In other words, they have signal dependent properties and result in presence of multiplicative behavior component in the obtained bispectral estimates.

Due to symmetry properties[16], the bispectrum estimate (2) is completely defined on the whole bispectrum plane *[p, q]* if it is computed in the principal triangle domain defined by inequalities $q \geq 0$, $p \geq q$, $p+q \leq I-1$. Taking into account a) and b), let us consider noise behavior on axes *q=0* and *p=q* in this principal triangular bifrequency domain:

$$\dot{N}(p,0) = \dot{S}(p)\left\langle \dot{N}^{(m)}(0)\dot{N}^{(m)*}(p)e^{-j\tau^{(m)}p}\right\rangle_M + \dot{S}(0)\left\langle \dot{N}^{(m)}(p)\dot{N}^{(m)*}(p)\right\rangle_M + \dot{S}^*(p)\left\langle \dot{N}^{(m)}(p)\dot{N}^{(m)}(0)e^{j\tau^{(m)}(p)}\right\rangle_M, q=0$$

$$\dot{N}(p,p) = \dot{N}(q,q) = \dot{S}(p)\left\langle \dot{N}^{(m)}(p)\dot{N}^{(m)*}(2p)e^{-j\tau^{(m)}p}\right\rangle_M + \dot{S}(p)\left\langle \dot{N}^{(m)}(p)\dot{N}^{(m)*}(2p)e^{-j\tau^{(m)}p}\right\rangle_M + $$

$$+\dot{S}^*(2p)\left\langle \dot{N}^{(m)}(p)\dot{N}^{(m)}(p)e^{j\tau^{(m)}(p)}\right\rangle_M, \qquad q=p$$

Because WGN is supposed to be delta-correlated, maximum noise contribution occurs for axes *q=0* and *p=q* due to concentration of WGN autocorrelation samples exactly in these axes (see the second term in expression for $\dot{N}(p,0)$ and the third term in latest formula for $\dot{N}(p,p)$). In practice of radars, the number *M* is limited, input SNR is usually low and the bispectrum estimates (3) are corrupted by noise and above mentioned distortions. Fluctuation errors are distributed in the whole 2-D bifrequency domain *[p,q]* and noise induced distortions are concentrated in axes *q=0* and *p=q*. Hence, due to presence of the noise terms in (3) the bispectrum estimates are biased.

Although there is no direct analytic dependence between the fluctuation errors in the reconstructed signal and the noise in bispectrum estimate, it is reasonable to expect that by obtaining better bispectrum estimates it is possible to decrease errors in the reconstructed signal. Hence, the problem arises, how to effectively perform noise suppression in bispectrum estimate (3) in order to minimize both fluctuation variance and bias in the reconstructed signal.

2.2. Conventional Approaches to Signal Reconstruction from Bispectrum Estimates

The conventional approaches to signal reconstruction[5,6,8] are based on the following fundamental equations that connect magnitude and phase bispectrum with signal Fourier magnitude and phase spectrum , respectively, as

$$\left|\dot{B}_S(p,q)\right| = \left|\dot{S}(p)\right|\left|\dot{S}(q)\right|\left|\dot{S}(p+q)\right|, \tag{5}$$

$$\gamma_S(p,q) = \varphi(p) + \varphi(q) - \varphi(p+q), \tag{6}$$

where $\left|\dot{S}(...)\right|$ and $\varphi(...)$ are the 1-D signal magnitude and phase Hermitian Fourier spectra, respectively, and $\dot{S}(p) = \dot{S}^*(-p)$; $\varphi(p) = -\varphi(-p)$.

The conventional algorithm of Bartelt et al[6] for signal amplitude Fourier spectrum recovery from magnitude bispectrum estimate can be described by the set of the following equations

$$\left|\hat{\dot{S}}(p+q)\right| = \frac{\left|\hat{\dot{B}}_x(p,q)\right|}{\left|\hat{\dot{S}}(p)\right|\left|\hat{\dot{S}}(q)\right|}, \quad p=0,...,I-1; \quad q=0,...,I-1;, \tag{7}$$

where $\left|\hat{\dot{S}}(...)\right|$ is the 1-D signal amplitude Fourier spectrum estimate that is recovered from the 2-D magnitude bispectrum estimate $\left|\hat{\dot{B}}_x(...,...)\right|$ (see (2)).

The recursive algorithm[6] for signal phase Fourier spectrum recovery can be written as follows

$$\hat{\varphi}(p+q) = \hat{\varphi}(p) + \hat{\varphi}(q) - \hat{\gamma}_x(p,q), \quad p = 0,...,I-1; \quad q = 0,...,I-1, \tag{8}$$

where $\hat{\varphi}(...)$ is the 1-D signal phase Fourier spectrum estimate that is recovered from the 2-D phase bispectrum estimate $\hat{\gamma}_x(...,...)$ (see the corresponding term in (2)). It should be noted, that $\hat{\varphi}(0) = 0$ in (8) because the considered signal is assumed to be real valued. The sample $\hat{\varphi}(1) = \hat{\varphi}(1) + \hat{\varphi}(0)$ is principally indetermined, hence, it is commonly assumed[6] that $\hat{\varphi}(1) = 0$.

Note the following important properties of the algorithms (7) and (8):
- each signal amplitude $\left|\hat{\dot{S}}(p)\right|$ and phase $\hat{\varphi}(p)$ Fourier spectrum estimate has *(p-1)/2* independent representations if *p* is odd and *p/2* representations if *p* is even;
- independent amplitude and phase representations offer different independent computation paths for recovery of the corresponding signal Fourier phase and amplitude spectra from bispectrum data, due to this redundancy[6] the additional averaging of the recovered signal 1-D Fourier spectrum magnitudes and phases takes place at recovery stage (7) and (8). But it should be specially stressed that noise reduction produced by the aforementioned averaging procedure is not the same for all samples of principal triangular bifrequency symmetry domain. Noise suppression efficiency for low frequencies is poorer than the noise smoothing that is achieved for high frequencies.

Finally, the signal reconstruction procedure can be represented as the following 1-D inverse Fourier transform (IFT)

$$\hat{s}_{rec}(i) = \left| IFT\{ \left|\hat{\dot{S}}(r)\right| e^{j\hat{\varphi}(r)} \} \right|, r = 0,...,I-1. \tag{9}$$

3. PROPOSED "BISPECTRUM-FILTERING" APPROACH TO SIGNAL RECONSTRUCTION

As known from the theory of linear and nonlinear filtering of noisy signals and images[18], the noise suppression is more effective and the introduced errors are smaller if the noise-free signal (image) is smooth slowly varying functions. In this sense, the original complex valued bispectrum $\dot{B}_s(p,q)$ can be such a function if the original signal is short pulse-like (an example of such signal is shown in Fig. 2). And this is a typical case for radar envelopes at detector output.

Then, one can see that there are many different ways to perform bispectrum estimate smoothing. Let us list some possibilities. First, the magnitude $\left|\hat{\dot{B}}_x(p,q)\right|$ and phase $\hat{\gamma}_x\ (p,q)$ bispectrum estimates (2) can be processed separately.

Second, the real $\mathrm{Re}\{\hat{\dot{B}}_x(p,q)\}$ and imaginary $\mathrm{Im}\{\hat{\dot{B}}_x(p,q)\}$ components can be also filtered. Then, the question is what is preferable? Besides, other important questions arise: 1) should the filtering be performed before or after ensemble averaging (2)? and 2) what filtering methods are the most efficient?

All this opens a wide space for future investigations. This is why, within the scope of this paper we consider only quite simple 2-D fixed size scanning window filters. The combined bispectrum-filtering methods assuming either smoothing $\left|\hat{\dot{B}}_x(p,q)\right|$ and $\hat{\gamma}_x\ (p,q)$ or processing $\mathrm{Re}\{\hat{\dot{B}}_x(p,q)\}$ and $\mathrm{Im}\{\hat{\dot{B}}_x(p,q)\}$ are studied and compared. The smoothing before and after bispectrum averaging (2) is considered. Besides, we pay emphasis to analysis of different input SNR values, including the cases of weak signals typical for radar applications.
Among a large number of modern linear and nonlinear filters the standard mean (linear) filter[18], the standard median[18], the FIR-median hybrid (FMH)[19] and the K-nearest neighbor (KNN)[20] filters have been chosen for statistical experiment and analysis of signal reconstruction system performance. All three latter filters are nonlinear. The selection of these four filters was motivated by the fact that they have different properties in the sense of noise suppression efficiency and detail preservation.

The choice of the scanning window size also influences the filtering efficiency[18]. Below we fixed the scanning window size to 5x5 samples since it is the most typical for 2-D image processing applications[18]. Recall that the output of the KNN filter is given as the mean value of the K_{NN} ($1 \le K_{NN} \le NxN$) samples whose values are the closest to the value of the scanning window central sample. The output of FMH filter (modification 3LH+ was used) is defined as the median value of the set of FIR subfilters[19].

The chosen types of filters listed above do not suppose the availability of a priori information about noise type (additive, multiplicative, mixed, etc.) and characteristics (for example, variance) like some other types of filters do (for instance, the standard sigma or the local statistic Lee filters). This is, at the same time, good and bad, since, on one hand, the selected filters can be easily applied to processing the noisy bispectrum estimates without making up preliminary analysis of their statistical properties. On the other hand, one can expect better performance for the case of more sophisticated filter application that take into account the characteristics of signal depending noise in bispectrum estimates (3). But this is a direction of further investigations in future.

4. DISCUSSION OF STATISTIC EXPERIMENT RESULTS

In this section the performance of the proposed combined "bispectrum-filtering" technique is compared to conventional bispectrum filtering of Bartelt et al[6] (BLW filter). We have to state, that the term "bispectrum-filtering" is used because the filtering procedures applied to $\left|\hat{\hat{B}}_x(p,q)\right|$ and $\hat{\gamma}_x\ (p,q)$ or $\mathrm{Re}\{\hat{\hat{B}}_x(p,q)\}$ and $\mathrm{Im}\{\hat{\hat{B}}_x(p,q)\}$ are integrated ("built in") into conventional BLW algorithm.

To assess the signal reconstruction system performance the following values have been analyzed:

a) The ensemble averaged WGN variance $\overline{\sigma}^2_{inp}$ and power $\overline{SNR}_{inp}$ at the signal reconstruction system input that are calculated as

$$\overline{\sigma}^2_{inp} = \frac{1}{M}\sum_{m=1}^{M}\sigma_G^{(m)2} , \tag{10}$$

$$\overline{SNR}_{inp} = P_s \big/ \overline{\sigma}^2_{inp} , \tag{11}$$

where $P_S = \frac{1}{I}\sum_{i=0}^{I-1}[s(i)-\bar{s}]^2$ is the power of the original signal and $\bar{s} = \frac{1}{I}\sum_{i=0}^{I-1} s(i)$ is the original signal mean value.

b) The output system ensemble averaged variance $\overline{\sigma}^2_{out}$ (random fluctuation errors at the signal reconstruction system output) that is defined as

$$\overline{\sigma}^2_{out} = \frac{1}{K-1}\sum_{k=1}^{K}\frac{1}{I}\sum_{i=0}^{I-1}[\hat{s}_k(i) - \overline{\hat{s}}(i)]^2 , \tag{12}$$

where $\hat{s}_k\ (i)$ is the k-th output reconstructed signal estimate; $\overline{\hat{s}}\ (i) = \frac{1}{K}\sum_{k=1}^{K}\hat{s}_k(i)$; K is the number of experiment repetitions employed for getting reliable estimate accuracy.

c) The MS bias (systematic error) $\delta^2{}_{out}$ at the output of signal reconstruction system derived as

$$\delta^2{}_{out} = \frac{1}{I}\sum_{i=0}^{I-1}[s(i)-\overline{\hat{s}}(i)]^2 . \tag{13}$$

The designed software[17] allows to model different input test signals, for example, as one or two short pulses with various shape(s), amplitude(s), mutual shifts of pulses and the pulse length(s). In this paper we pay main attention to investigation of the test model in the form of two short radar pulses (see Fig. 2) with rectangular shapes and different amplitudes. The pulse amplitudes are equal to $A1$=3 (this pulse can correspond, for example, to higher power malicious interference or clutter that often mask a useful information signal) and $A2$=1 (assume that this pulse corresponds to the information radar response). This two-peak model is also used for target range profile description. The pulse lengths are

$\Delta t1=\Delta t2=3$ samples. The mutual shift of these pulses is $\Delta t_{12} = 6$ samples. The power of this original noise-free signal is equal to $P_s \approx 0.558$.

The original (noise-free) test deterministic signal *s(i)* used in our statistical experiment is simulated as the sequence of non-negative real values generated for a 1-D array of I=256 samples ($i=0,1,...,255$). This original deterministic signal component is repeated for each realization while the random additive WGN component and random original signal shift $\tau^{(m)}$ are generated each time again for every new Monte Carlo run employed in averaging procedure (2).

The corruption sources have been simulated in the following way: WGN $n_G(i)$ had the fixed variance (that was constant within one case experiment but varied for getting the performance dependence upon $\overline{SNR}_{inp}$) and with zero mean. The random shift $\tau^{(m)}$ of the original signal was supposed to have uniform distribution with the fixed maximal deviation of ±12 samples. One realization of the test randomly shifted original 1-D signal corrupted by additive WGN with $\sigma_G^{(m)2} \cong 0.56$ and $\overline{SNR}_{inp} \cong 1.0$ (only non-negative signal values are displayed in the plots) is represented in Fig. 3.

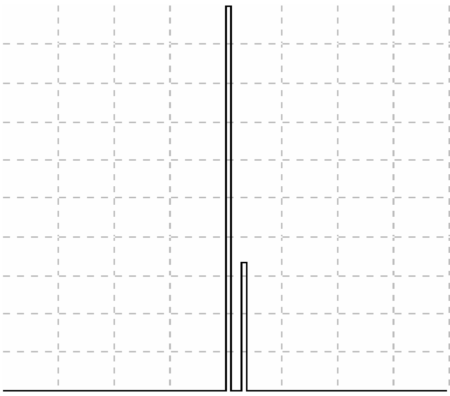

Fig.2. The noise-free test signal.

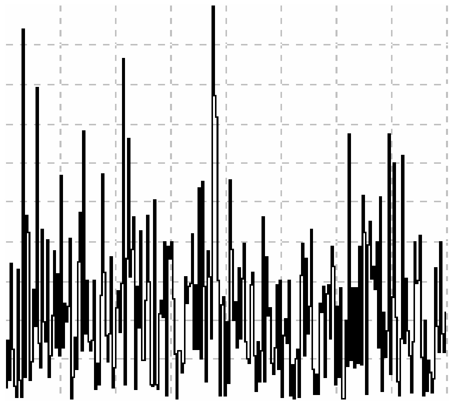

Fig. 3. The *m*-th realization of normalized test signal corrupted by additive WGN ($\overline{SNR}_{inp}$ =1.0) and random shift.

The following new different ways of bispectrum estimates smoothing by scanning window filtering are investigated:

- smoothing each *m*-th magnitude $\left|\hat{\hat{B}}_x^{(m)}(p,q)\right|$ and phase $\hat{\gamma}^{(m)}{}_x(p,q)$ bispectrum estimate realization before the averaging procedure (2);
- smoothing averaged magnitude $\left|\hat{\hat{B}}_x(p,q)\right|$ and phase $\hat{\gamma}_x(p,q)$ bispectrum estimates (see the formula (2));
- smoothing real $\mathrm{Re}\{\hat{\hat{B}}_x^{(m)}(p,q)\}$ and imaginary $\mathrm{Im}\{\hat{\hat{B}}_x^{(m)}(p,q)\}$ components of each *m*-th bispectrum estimate realization $\hat{\hat{B}}_x^{(m)}(p,q)$ before the averaging procedure (2);
- smoothing averaged real $\mathrm{Re}\{\hat{\hat{B}}_x(p,q)\}$ and imaginary $\mathrm{Im}\{\hat{\hat{B}}_x(p,q)\}$ bispectrum estimate components.

The investigation results demonstrate that the restoration system performance weakly depends upon whether the smoothing is carried out “before” or “after” averaging (2). At the same time, the performance essentially depends upon do we process $\left|\hat{\hat{B}}_x(p,q)\right|$ and $\hat{\gamma}_x(p,q)$ or $\mathrm{Re}\{\hat{\hat{B}}_x(p,q)\}$ and $\mathrm{Im}\{\hat{\hat{B}}_x(p,q)\}$. The smoothing of the real and imaginary bispectrum components by 2-D filters provides sufficiently better performance as compared to processing of

$\left|\hat{\dot{B}}_x(p,q)\right|$ and $\hat{\gamma}_x(p,q)$. For all the considered 2-D filters the better output system ensemble averaged variance (12) and the smaller MS bias values (13) are provided for the case of smoothing Re{ $\hat{\dot{B}}_x(p,q)$ } and Im{ $\hat{\dot{B}}_x(p,q)$ }. This is due to the fact that both $\left|\hat{\dot{B}}_x(p,q)\right|$ and $\hat{\gamma}_x(p,q)$ (in contrast to Re{ $\hat{\dot{B}}_x(p,q)$ } and Im{ $\hat{\dot{B}}_x(p,q)$ }) contain discontinuities since they are obtained by nonlinear transformations of complex valued bispectrum $\hat{\dot{B}}_x(p,q)$. In the neighborhoods of discontinuities any kind of 2-D filter introduces considerable distortions. And this results in increasing both $\overline{\sigma}^2_{out}$ and δ^2_{out}.

To get an idea what is noisy bispectrum estimate, the real and imaginary components $\hat{\dot{B}}_x(p,q)$ are represented in Figures 4 and 5, respectively. As seen, both real and imaginary bispectrum estimate components are drastically destroyed due to leakage of WGN from reconstruction system input. Note, that in accordance to formula (4), noise is the most intensive for bifrequency axes p=0, q=0 and p=q (see Fig. 4).

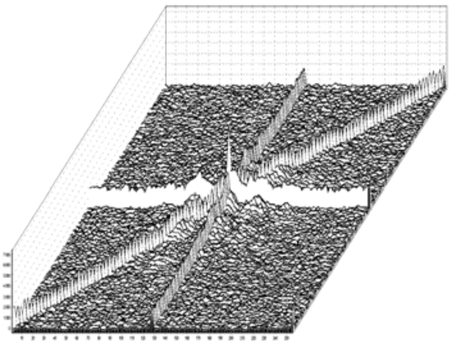

Fig. 4. Noisy real bispectrum component ($\overline{SNR}_{inp}$ =1.0).

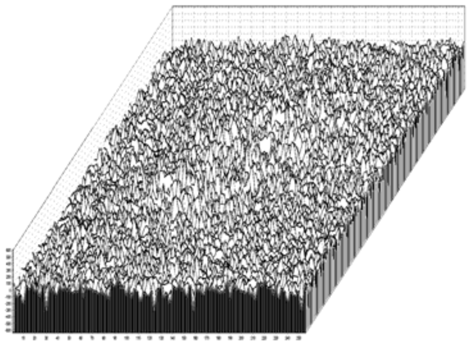

Fig. 5. Noisy imaginary bispectrum component ($\overline{SNR}_{inp}$ =1.0).

The real (see Fig. 6) and imaginary (see Fig. 7) bispectrum components smoothed by 2-D mean filter still contain noise. But its level is sufficiently less (see for comparison Figures 4 and 6, 5 and 7, respectively).

The values $\overline{\sigma}^2_{out}$ (12) and δ^2_{out} (13) for conventional BLW method[6] and the proposed combined bispectrum-filtering techniques have been computed for a set of $\overline{SNR}_{inp}$. In simulations we use M = 256 realizations repeated K=30 times. This provides the required accuracy of numerical simulation for processing statistical data as well as satisfactory reconstructed signal estimate consistency[15].

The plots of $\overline{\sigma}^2_{out}$ (12) and δ^2_{out} (13) depending upon $\overline{SNR}_{inp}$ are represented in Figures 8 and 9, respectively. The analysis of the plot in Fig. 8 shows that the fluctuation variances tend to negligibly small values with $\overline{SNR}_{inp}$ increasing starting from $\overline{SNR}_{inp}\geq$5. The use of the mean, KNN and FMH filtering decrease $\overline{\sigma}^2_{out}$ in comparison to

BLW method, the best result is provided by 2-D mean and KNN filters. Note, that considerable benefit is observed for low $\overline{SNR}_{inp}$. The worst result in the sense of largest $\overline{\sigma}^2_{out}$ is observed for the case of median filter application.

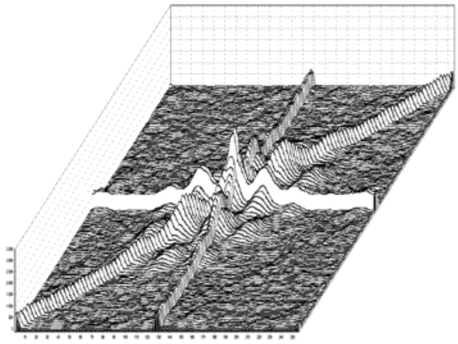

Fig. 6. Real bispectrum component ($\overline{SNR}_{inp}$=1.0) processed by the 5x5 standard mean filter.

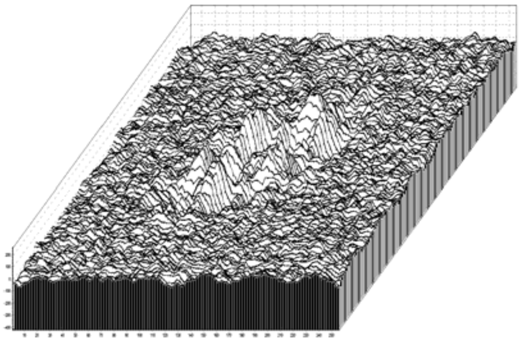

Fig. 7. Imaginary bispectrum component ($\overline{SNR}_{inp}$=1.0) processed by the 5x5 standard mean filter.

The analysis of the plots δ^2_{out} vs $\overline{SNR}_{inp}$ (see the Fig. 9) permits to conclude the following. First, the reconstructed signal estimates as it could be predicted from theoretical analysis in Section 2 are biased. The bias values decrease with $\overline{SNR}_{inp}$ increasing. The mean, KNN and standard median filters provide δ^2_{out} improvement in comparison to BLW method. The best result is again provided by the standard mean filter ensuring the minimum MS bias. Despite the reconstructed signal estimates are biased, the bias values are relatively small. It might seem that the presence of bias may cause problems in signal shape or signal parameter estimating. However, to our opinion, it does not lead to serious restrictions in such radar/sonar practical applications as, for example, object identification in noise, object resolution under influence of malicious interference, etc.

In aggregate, the use of the 5x5 standard mean filter produces the best results among the considered filters for given signal model and wide range of $\overline{SNR}_{inp}$. Both $\overline{\sigma}^2_{out}$ and δ^2_{out} have been decreased by approximately two times in comparison to BLW method for low $\overline{SNR}_{inp}$ that is the most important case in radar/sonar applications.

The visual analysis results are presented in Figures 10 and 11. The normalized plot of signal reconstructed from bispectrum estimate smoothed by 5x5 standard mean filter is shown in Fig. 10, and for BLW method in Fig. 11. Despite the small object amplitude is decreased (Fig. 10) and the reconstructed signal is a little bit distorted in comparison to original signal (Fig. 2), the reliable identification of the small intensity object *A1* is provided on the background of suppressed noise. It is clearly seen that the noise suppression efficiency of the proposed bispectrum-filtering method is better than for BLW one (compare the Figures 10 and 11).

5. CONCLUSIONS

The combined "bispectrum-filtering" approach for improving the performance of unknown signal waveform reconstruction system has been proposed and investigated. The novel approach is based on smoothing the noisy estimate data by 2-D linear and nonlinear filters. Several proposed hybrid “bispectrum-filtering” algorithms that are rather robust to additive WGN have been designed, investigated and compared both to each other and to the conventional bispectrum-based signal reconstruction technique. Numerical simulation results demonstrate high robustness to interference of the proposed combined bispectrum-filtering techniques in the sense of effective suppression of both intensive additive WGN noise and insensitivity to random signal shifts. The results obtained for fluctuation variances and MS bias show that the filtering of real and imaginary components of bispectrum estimates is more efficient than the processing of magnitude and phase bispectrum estimates in the sense of obtaining more reliable bispectrum estimates and, hence, of providing better signal reconstruction system accuracy and performance. The reconstruction system performance weakly depends upon the smoothing stage "before" and "after" ensemble average. The proposed techniques incorporate the advantages of both bispectral processing and linear/non-linear filtering. They can be recommended for cases of weak signal processing and small input SNRs typical for radar and sonar applications.

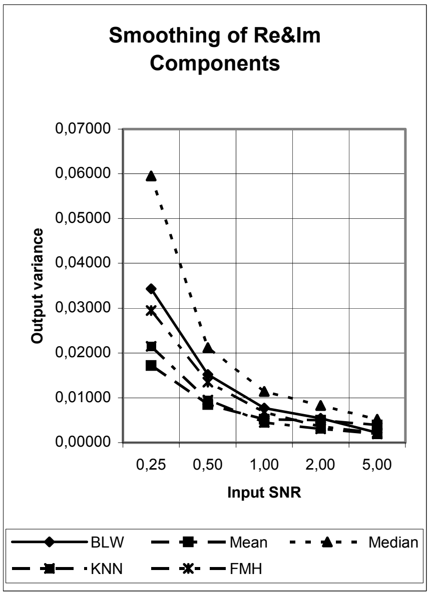

Fig. 8. Output variance $\overline{\sigma}^{2}_{out}$ vs $\overline{SNR}_{inp}$.

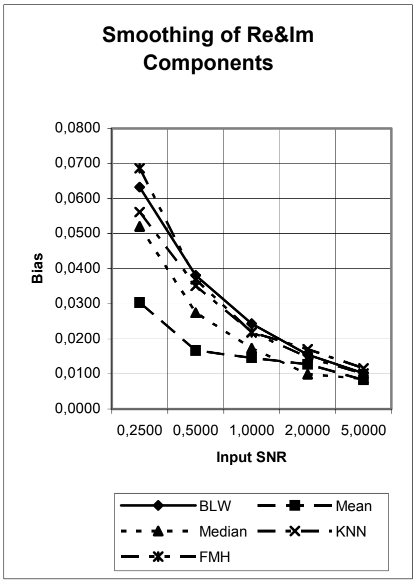

Fig. 9. MS bias vs $\overline{SNR}_{inp}$.

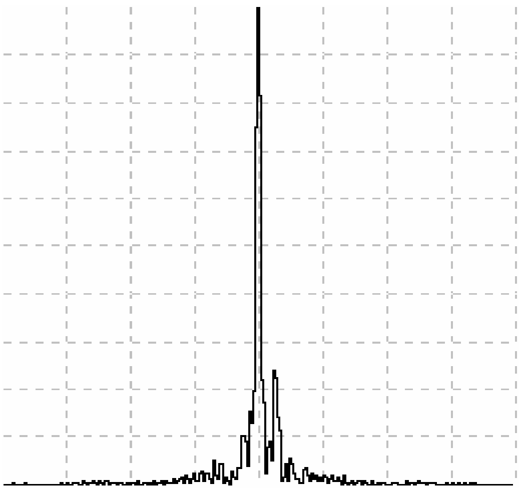

Fig. 10. The normalized signal reconstructed from bispectrum estimate for which the Re and Im components have been smoothed by 5x5 Mean filter ($\overline{SNR}_{inp}$ =1.0).

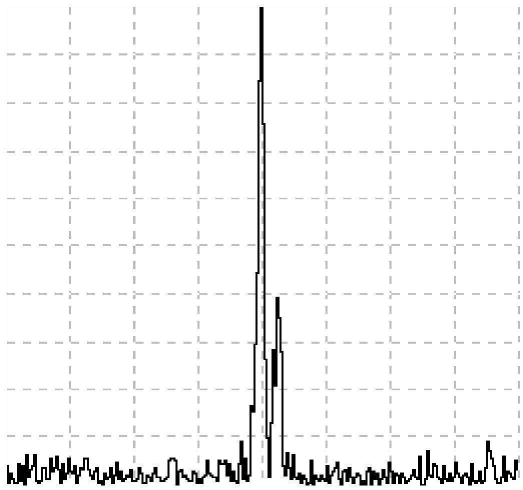

Fig. 11. The normalized signal reconstructed from bispectrum estimate by conventional BLW technique for $\overline{SNR}_{inp}$ =1.0.

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