

Testing of methods for blind estimation of noise variance on large image database

Vladimir V. Lukin^a, Sergey K. Abramov^a, Mikhail L. Uss^{a,b}, Igor Marusiy^a, Nikolay N. Ponomarenko^a, Alexander A. Zelensky^a, Benoit Vozel^b, Kacem Chehdi^b

^aNational Aerospace University, 61070, Kharkov, Ukraine;

^bUniversity of Rennes I, Lannion, France

1. Introduction

Currently there is an obvious need in applying blind (automatic), accurate, reliable (robust) and fast methods and algorithms for estimation of noise type and characteristics, at least, variance or standard deviation for images at hand. This is needed for optical grayscale and color images (see [1, 2] and references therein), radar images [3, 4], multispectral and hyperspectral remote sensing (RS) data [5, 6], video and surveillance [7], etc.

Determination of noise type and evaluation of noise characteristics is desirable since original images formed by different types of systems are usually noisy due to various phenomena and noise statistics is often not known in advance and/or can change in an unpredictable way. At the same time, many image processing methods intended for filtering, reconstruction, edge detection, segmentation, and compression require a priori knowledge on noise type and statistics [1]. Thus, noise statistics is often to be estimated for an image at hand.

In general, sometimes this can be done in interactive manner if a highly qualified expert has the corresponding software at disposal. However, first, this cannot be done always when needed. For example, such estimation is impossible if noise is to be analyzed on-board of a spaceborne carrier of a remote sensing system. It is also impossible if such estimation has to be carried out very quickly as in video and surveillance applications. Second, estimation becomes very complicated and/or labor-consuming if amount of images or its components is large and processing has to be performed operatively. Such situations take place in multi- and hyperspectral remote sensing when each image contains tens or hundreds of sub-band components and RS data are exploited for ecological monitoring, catastrophe prediction, flood control and other similar purposes [8].

Note that image pre-processing (pre-filtering, reconstruction, etc.) that relies on pre-estimated type and parameters of noise and other degradations is often required in practice. For example, in many cases it leads to better classification of RS data [9, 10]. It might result in better visual quality of images [11, 12] and so on.

One can take advantage of the fact that blind methods for determination (identification) of noise type have been already proposed [1, 13-15]. It is possible to determine noise type and to estimate noise characteristics jointly or as separate stages of image analysis [1, 13]. As it has been shown in experiments [13, 16], correct determination of noise type can be carried out with rather high probability. Thus, it is possible to concentrate on estimating noise characteristics for a given type of noise.

Methods for blind evaluation of noise characteristics, mostly variance, were started to be designed in early 90-th of the previous century [3, 4, 17]. Let us explain what is meant by the aforementioned requirement to them to be robust. In many applications, it is unknown in advance what can be such characteristics of noise to be met in a given image as, e.g., noise variance (or SNR). It is often not known a priori is noise spatially correlated or not. Moreover, image structure (how many details, edges, textures, homogeneous regions an image contains) is also frequently unknown in advance even approximately. However, a given method for noise characteristic estimation should perform reasonably well for a wide class of images at hand, different level (variance) and spatial correlation of noise, i.e. it has to be robust in wide sense according to P. Huber's definition. Examples of such lack of robustness for the method [18] are demonstrated in the paper [19]. It is shown that the method [18] produces large bias of additive noise variance evaluation if noise is spatially correlated. Considerable efforts have been also spent on providing appropriate accuracy of blind methods of variance evaluation in textured images [20, 21] that are the most complex (unfavorable) case of image structure for the considered task.

One question is what is appropriate accuracy? It has been demonstrated [22, 23] that if peak signal-to-noise ratio (PSNR) in an image is within the limits 20...34 dB, then it is desirable to provide an estimate of (additive or multiplicative) noise variance within the limits of 0.8...1.2 of its true value. In this sense, there are various ways to characterize a method accuracy and robustness. For example, a method accuracy can be described by some conventional statistical parameters as mean (or bias) and standard deviation (or variance) of noise variance estimates obtained for an appropriately large set of noise

realizations and test images. Taking into account that estimate distribution for this approach is not Gaussian (see data in Section 4) and keeping in mind above mentioned practical requirement to a method accuracy, we prefer to analyze how often blind estimates occur to be out the limits 0.8...1.2 of noise variance true value. This approach also gives imagination about robustness of a method since the fact of noise variance estimate out of the limits can be considered as inappropriate robustness.

However, a problem is that methods described in literature are commonly tested on few images (at least, the results are presented in this way, probably, due to lack of space). These images are either artificially synthesized or they are the standard ones in image processing like Lena, Barbara, Baboon, House, etc. Sometimes, one or two real life images are processed as examples of method applicability but then it is hard to establish how accurate the obtained estimates are.

To partly fill this gap, we have decided to test some methods for blind evaluation of noise variance for considerably greater number of natural images. Note that recently a database TID2008 was created [11, 24]. This database contains 25 color images (mostly taken from Kodak image database <http://r0k.us/graphics/kodak/>) corrupted by 17 types of distortions each with 4 levels. Two types of distortions are spatially uncorrelated and spatially correlated additive noises. This allows testing any blind method for 150 images corrupted by additive noise with the same variance (R, G, and B components of each of 25 test images with either i.i.d. or spatially correlated noise). In turn, one can then analyze robustness of a considered method to image content, average accuracy of a method. It also becomes possible to find image and noise situations for which a considered method performs poorly and so on. This is the main goal of this paper.

The paper structure is the following. In Section 2, we briefly describe the image set in the database TID2008 and discuss how noise was simulated. Section 3 deals with considering the blind methods for additive noise variance evaluation used in our study. Section 4 presents the main results of this study and analysis of the considered methods' accuracy. The reasons why the required accuracy is not provided in some cases are discussed. Finally, the conclusions are drawn.

2. Test image set and noise characteristics

TID2008 contains 25 noise-free (high quality) color images where first 24 are the fragments of original Kodak database and the 25-th image is the artificial image with different textures (see Fig. 1). As seen, the images are quite different; there are natural scenes, portraits, houses, etc. All color images are RGB 24-bit ones.





Fig. 1. Test noise-free color images of TID2008

All images in the database are of size 512x384 pixels. This choice has been suggested for unification purpose (the images in the Kodak test set have non-equal sizes) and for convenience of carrying out subjective experiments (see details in [11, 24]).

According to the methodology of subjective experiments proposed in [11] and intended for analysis of visual quality of images distorted by different types of degradations, it was necessary to provide four values of PSNR approximately equal to 21, 24, 27, and 30 dB. For additive noise it was not a problem, a required PSNR of a distorted image can be easily produced by simulating noise with such variance that

$$PSNR_{req} (dB) = 10 \lg(255^2 / \sigma^2), \quad (1)$$

where σ^2 is variance of noise. Therefore, noise variance had to be 65 (for $PSNR_{req}=30$ dB), 130 ($PSNR_{req}=27$ dB), 260 ($PSNR_{req}=24$ dB), and 520 ($PSNR_{req}=21$ dB).

In our studies described in this paper we do not consider all these values of noise variance. The first two values (65 and 130) are of more practical interest since two other ones (260 and 520) are rarely met in practice of 8-bit image processing. Besides, for most methods of blind evaluation of additive noise variance it is more difficult to provide appropriate accuracy of estimation for smaller values of noise variance than for large ones [20].

Thus, let us below concentrate on analysis of images corrupted by noise with variances 65 and 130. Availability of TID2008 (<http://www.ponomarenko.info/tid2008.htm>) allows an interested researcher to carry out his/her own experiments for other sets of images.

Obviously, simulation of i.i.d. noise is not a problem at all. Spatially correlated noise with variances given above has been simulated in a simple way by first applying the 3x3 mean filter to an array of i.i.d. noise and then adjusting a required variance to the obtained noise. One can argue that this is only one particular case of spatially correlated noise. This is really so. However, a reader should keep in mind that our purpose was only to verify performance of the considered blind methods for spatially correlated noise case in order to know does a method fail to work well or no. More details concerning performance of several methods can be found in the paper [19].

Note that spatially correlated noise (with the same variance as i.i.d. noise) has more unpleasant appearance [25]. The corresponding images are perceived as having worse visual quality (compare the image in Fig. 2,b to the image in Fig. 2,a). Moreover, spatially correlated noise is much more difficult to filter out [26].



Fig. 2. The same image corrupted by i.i.d. (a) and spatially correlated noise (b) with variance 130

Certainly, it was possible that a simulated value $I_{ij}^n = I_{ij}^{true} + n_{ij}$ occurred to be out of the limits $0, \dots, 255$ where I_{ij}^{true} is the true value in the ij -th pixel of a given component of RGB image, I_{ij}^n is the noisy value, n_{ij} denotes simulated additive noise. Then we returned the simulated values back to the limits $0, \dots, 255$ by assigning the closer limit value to keep 8-bit representation of data. This could slightly change a practically achieved PSNR in comparison to the corresponding required one, but such “saturation” corresponds to practice, e.g., how this is done to fit image data to pre-determined limits [27]. Here we mention clipping effects since, as it will be shown below, they might lead to some specific problems in blind estimation of noise variance.

Also note that noise has been modeled as independent for different components of RGB color image. This also corresponds to practice [2, 28].

The method of noise type blind identification [13] has been applied to all images (totally, 600 ones since identification of noise type has been performed component-wise [1] for all 25 test images corrupted by four values of noise variance, both spatially uncorrelated and correlated). For all considered images the noise has been identified as additive. This demonstrates very good performance of the method [14] and its practical applicability, at least, for images corrupted by additive noise. In future, we plan to test the method [14] to images of TID2008 corrupted by other types of noise and degradations.

3. The used blind methods for evaluation of additive noise variance

It is impossible to consider all known methods for blind evaluation of additive noise variance (see [1] and references therein). Because of this, let us analyze several particular methods that belong to different groups.

One, probably the largest, group includes the methods operating in spatial domain [29]. The methods that belong to this group are based on assumption that blocks of a certain size tessellate an image and there is a set of blocks that belong to image homogeneous regions. The local estimates of noise variance obtained for these blocks are quite close to a true value of noise variance. These “normal” local estimates form a distribution mode that can be found (estimated).

An example of histograms of local variance estimates obtained for the known (standard) test image Barbara corrupted by spatially correlated additive noise with variance 100 for non-overlapping blocks of three different sizes is presented in Fig. 3. As it is seen, they all have maximum (mode) in the neighborhood of the true values of noise variance although the positions and widths of these maximums depend upon the block size (the width is the smallest for 9×9 blocks). Besides, all distributions (characterized by their histograms) have heavy right-hand tails (these tails are depicted not totally, considerably larger values of local estimates have been observed). These large (abnormal) local estimates are obtained for image heterogeneous blocks that correspond to edges, details and texture.

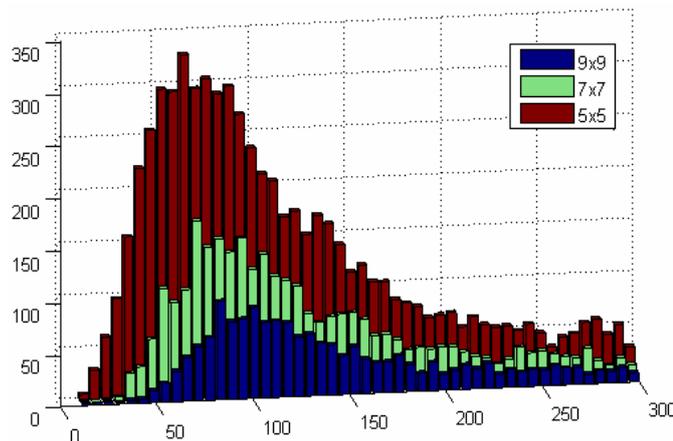


Fig. 3. Examples of histograms of the local estimates $\{\hat{\sigma}_k^2, k = 1, \dots, K\}$ (K is the number of blocks used) for non-overlapping blocks for noisy image Barbara corrupted by spatially correlated noise

There can be particular differences for the considered group of methods in how to estimate local variances, how to find the distribution mode (this principle is put into basis of the considered group of methods), to pre-process an image or not. However, the general idea of these methods works well enough if there are enough image homogeneous blocks [1, 19, 21, 29, 30].

Among the methods that belong to this group we have decided to test the method [21] in the first order. This method is quite simple and fast; it exploits finding minimal inter-quantile distance of sorted

(in ascending order) local estimates of noise variance for finding a preliminary estimate $\hat{\sigma}_{prn}^2$. Then the estimates in the neighborhood of preliminary estimate are approximated and a final estimate of distribution mode $\hat{\sigma}_n^2$ is found.

Note that if an analyzed image is rather large, i.e., contains hundreds of thousands of pixels, a difference in final estimates obtained for different realizations of noise with the same variance is relatively small, commonly considerably smaller than estimation bias absolute value [19]. This allows analyzing data for one or few realizations of noise to get imagination on provided accuracy of blind evaluation.

Another group of methods is based on exploiting difference of image content and noise in spatial spectrum domain [1, 18, 20, 31]. The basic idea is that noise is spread among all spectral components (if noise is i.i.d., it is spread uniformly) whilst information is mainly contained (concentrated) in a limited number of spectral components. Wavelets [18] and DCT [20] as well as other orthogonal transforms can be used for obtaining and further processing of data in spectral domain. Processing of spectral coefficients has to be done so that large amplitude spectral components are practically neglected (considered as outliers). This means that robust data processing methods are to be applied.

The methods of this group, especially [20, 31], perform rather well even for very textural images, but their common drawback is that this takes place only if noise is i.i.d. or it exhibits very small spatial correlation. For the methods [18, 20] some results will be presented in Section 4.

Recently, methods based on maximum likelihood (ML) estimation of noise and image parameters have been proposed [32, 33]. The main goal in designing these methods is to achieve better performance for highly textural images. For wavelets or DCT-based methods to operate well one main condition should be satisfied: standard deviation (SD) of a number of texture high-spectral coefficients has to be small compared to the noise standard deviation. Then it is possible to detect (select) these coefficients and use them to estimate noise variance. However, this condition can be violated for highly textural images and/or low noise level. The condition can be softened if we can predict and eliminate texture high-spectral coefficients based on low- and middle-spectral ones. This was done in [32, 33] by introducing texture parametric model, namely 2D fractal Brownian motion (fBm) model. This model can be locally adjusted in statistical sense to the texture with respect to texture amplitude and roughness. Then, noise variance estimation task is stated as maximum likelihood problem of joint estimation of texture parameters (locally) and noise variance (globally). This approach has proved to be effective for uncorrelated additive and multiplicative noise variance estimation [32, 33]. But it can lead to biased estimation when texture model is significantly different from fBm-model. Detection of such differences is of great importance for providing reliable noise variance estimation.

If noise is correlated, then its correlation matrix should be introduced in ML scheme, otherwise noise variance estimation will be biased. In this paper, to cope with unknown noise correlation structure, the following modification is introduced: the noise is made spatially uncorrelated by resampling (downsampling) an original image under processing. In practice, it is commonly enough to use downsampling by a factor 2...3 for both directions (taking into account that noise correlation length is commonly less than 1-2 pixels in practice). After this, the ML method [32, 33] for blind noise variance estimation can be applied. In order to provide comparable results, the modified version of the ML algorithm is used below for estimating both correlated and uncorrelated noise variance.

4. Accuracy analysis of the considered methods

4.1. Variance estimation for spatially uncorrelated noise

Let us consider the results obtained for the method [21]. Fig. 4 presents the plots for red, green, and blue components of the test images for noise variance 65. The block size is 5x5 pixels. Non-overlapping blocks have been used.

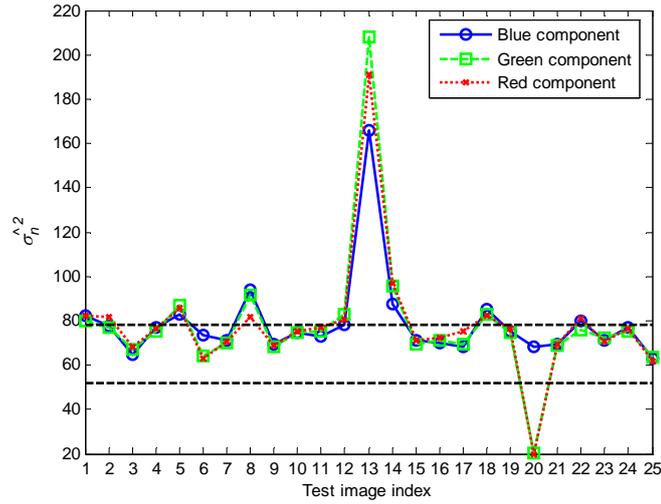


Fig. 4. Noise variance estimates obtained by the method [21] with nonoverlapping 5x5 pixels block size for red, green and blue components for the test image set corrupted by spatially uncorrelated additive noise with $\sigma_n^2=65$

There are several observations that follow from Fig. 4:

- 1) in most cases, the estimates $\hat{\sigma}_n^2$ obtained for different components of the same color image are quite close; the only exception is the 20-th test image; the reason will be analyzed later;
- 2) in most cases, the estimates $\hat{\sigma}_n^2$ are larger than the true value of variance (equal to 65); the only exception is the 25-th test image as well as green and red components of the 20-th test image;
- 3) although $\hat{\sigma}_n^2$ are commonly larger than σ_n^2 , the estimates $\hat{\sigma}_n^2$ are mainly within the required limits from $65 \times 0.8 = 52$ till $65 \times 1.2 = 78$ (the required limits are indicated by two horizontal lines in Fig. 4 and later in other plots); the exceptions are the test images 1, 5, 8, 12, 13, 14, 18, and 22; note that all these images are either textural (especially the 13-th image which is extremely textural) or contain many details as the 5-th and 8-th images.

Let us consider the same blind estimation method but for 7x7 blocks. The results are presented in Fig. 5. The conclusions are practically the same as for the plots in Fig. 4. The difference of the estimates for blocks of sizes 5x5 and 7x7 is not large. This means that for spatially uncorrelated noise there is practically no difference in the block size 5x5 or 7x7.

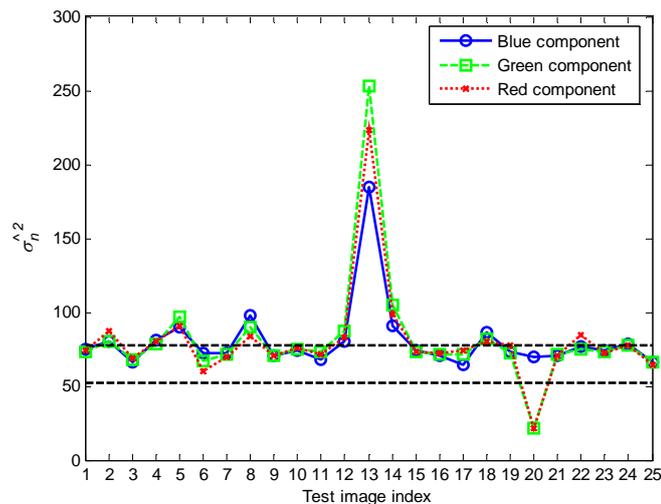


Fig. 5. Noise variance estimates obtained by the method [21] with nonoverlapping 7x7 pixels block size for red, green and blue components for the test image set corrupted by spatially uncorrelated additive noise with $\sigma_n^2=65$

Let us analyze what happens to the green and red components of the 20-th test image. The histogram of local variance estimates for red component is depicted in Fig. 6. As it is seen, it really has maximum for the argument about 20 (another maximum corresponds to the argument about 65 but its amplitude is smaller). Thus, the mode determination algorithm that finds the largest maximum coordinate performs correctly. The reason why quite many local estimates are considerably smaller than the true value of noise variance deals with clipping effects. In the red and green components of the original noise-free 20-th image, there are many positions of blocks (placed in the upper part of the image that corresponds to sky, see Fig. 1) for which the block means are close to 255. Thus, after adding simulated noise and returning the obtained values into the limits from 0 to 255 quite many “noisy” values occur to be equal to 255. Then, the local variance estimates for the corresponding blocks are “distorted” (smaller than they should be) due to such “clipping”.

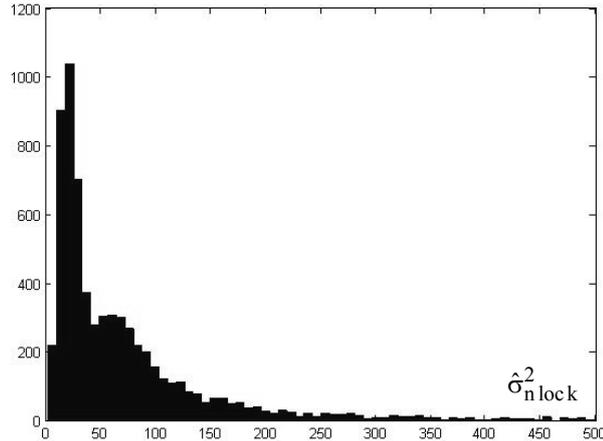


Fig. 6. Histogram of local estimates for the red component of the 20th test image

The clipping effect has been already mentioned and considered in the paper of A. Foi [34]. Note that the effects of clipping might happen in practice due to different reasons [34]. Their negative influence on the final estimate $\hat{\sigma}_n^2$ demonstrated above means that local variance estimates obtained in blocks where clipping is observed should be removed from analysis before the algorithm for determination of $\hat{\sigma}_n^2$ is applied. For example, it is possible to control how many values I_{ij}^n in a given block are equal to 255 or 0. If their amount N_{clip} is larger than $\beta_{clip} S_{bl}$ (where S_{bl} is the number of pixels in one block, β_{clip} is a preset parameter), then a given block is removed from further consideration. A rough recommendation is to set $\beta_{clip} \approx 0.15$. However, this recommendation needs additional verification.

Consider another value of noise variance, namely, 130. The obtained results are presented in Fig. 7 (the required limits are 108 and 156 for $\sigma_n^2 = 130$). The block size is 5x5 pixels. First, it is possible to compare the estimates $\hat{\sigma}_n^2$ in Fig. 4 (for $\sigma_n^2 = 65$) and in Fig. 7 (for $\sigma_n^2 = 130$). Such comparison shows that for a given particular test image and the same color component the estimates $\hat{\sigma}_n^2$ in Fig. 7 are almost twice larger than in Fig. 4. Thus, all conclusions drawn from analysis of the plots in Fig. 4 are also valid for the plots in Fig. 7. Again the estimates are quite accurate except the estimates for several images mentioned earlier which are either textural or for which clipping effects are observed.

The results for 7x7 blocks are similar to those ones presented in Fig. 7.

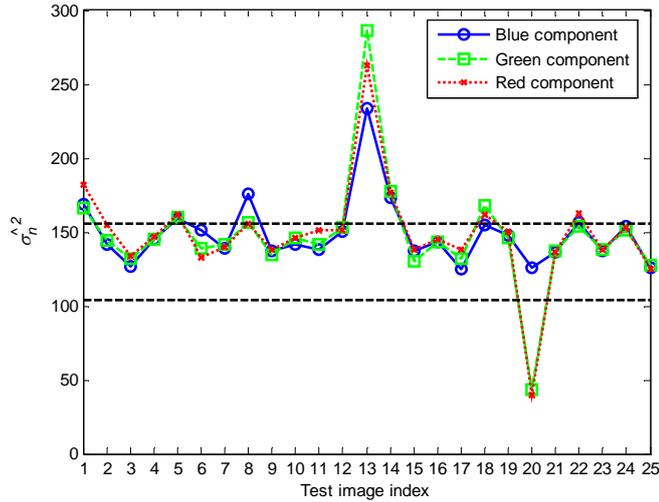


Fig. 7. Noise variance estimates obtained by the method [21] with nonoverlapping 7x7 pixels block size for red, green and blue components for the test image set corrupted by spatially uncorrelated additive noise with $\sigma_n^2=130$

Let us present some results for the method [18] that belongs to the second group of blind methods. Fig. 8,a shows the estimates obtained for i.i.d. noise with variance equal to 65. In general, they are similar to those ones presented in Figures 4 and 5. Larger estimates are observed for images that are more textural. Almost all estimates are larger than the true value. Approximately half of them are larger than the upper bound of the limits considered appropriate (52...78). There are two exceptions when the estimates are smaller than the lower bound. They are observed for green and red components of the 20th test images for which the clipping effects have impact on the final estimates. It seems that clipping effects cannot be easily taken into account in the method [18].

Fig. 8,b shows the estimates for the case of i.i.d. noise with $\sigma_n^2 = 130$ for the method [18]. They are at similar level as those ones presented in Fig. 7. Again, large estimates are observed for textural images 1, 5, 8, 13, 14, 18. Meanwhile, there are quite many estimates within the required limits from 104 to 156. Due to clipping effects, the estimates for the green and red components of the 20th test images are considerably smaller than they should be.

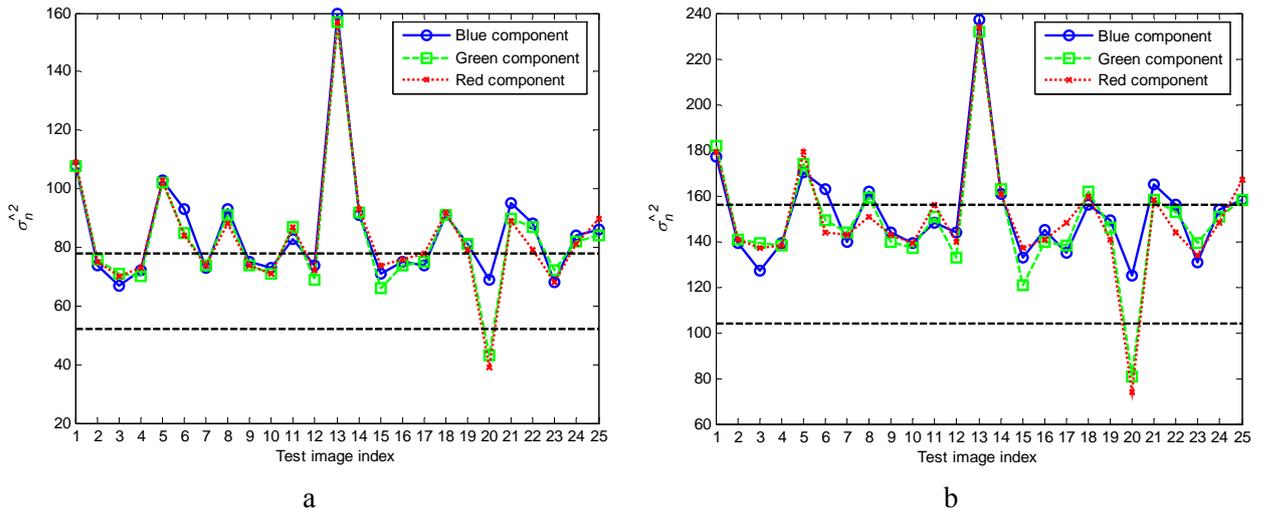


Fig. 8. Noise variance estimates obtained by the method [18] for red, green and blue components for the test image set corrupted by spatially uncorrelated additive noise with $\sigma_n^2=65$ (a) and $\sigma_n^2=130$ (b)

The method [20] modified in [31] for which removal of clipped areas has been done produces considerably more accurate estimates. They are presented in Fig. 9,a for noise variance equal to 65 and in Fig. 9,b for $\sigma_n^2 = 130$. As seen, only for two components of the highly textural 13th test image the estimates are out the required limits but only slightly.

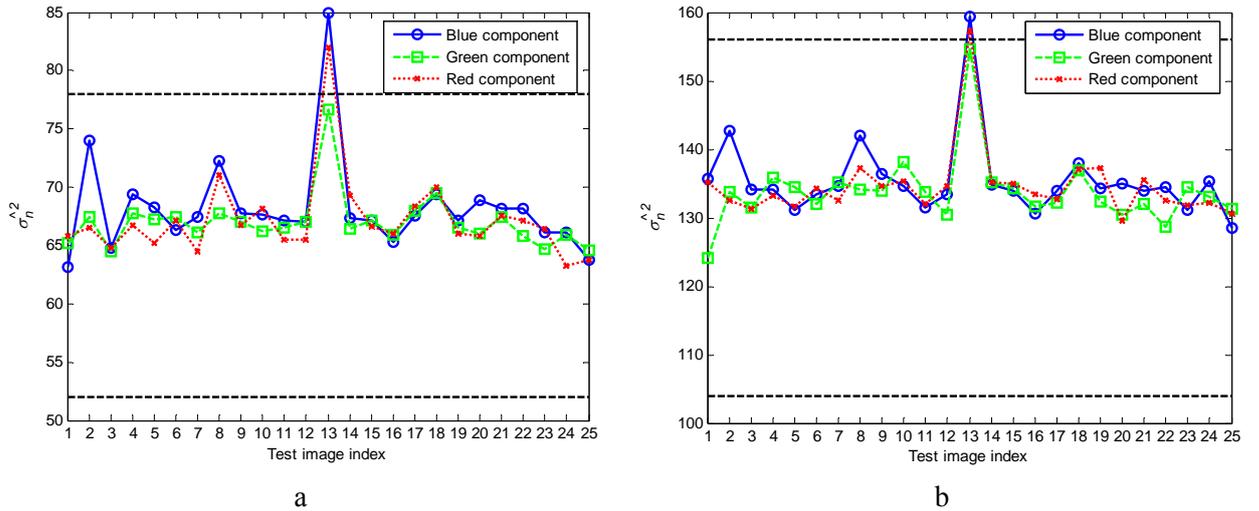


Fig. 9. Noise variance estimates obtained by the method [31] for red, green and blue components for the test image set corrupted by spatially uncorrelated additive noise with $\sigma_n^2=65$ (a) and $\sigma_n^2=130$ (b)

Consider now the methods based on maximum likelihood estimation of noise and image parameters [32, 33]. Since here we are not interested in results of image parameters' estimation, let us concentrate on the obtained estimates of noise variance. Data for $\sigma_n^2=65$ are presented in Fig. 10.a. Fragments with clipping effects have been preliminarily removed.

As it is seen, the estimates are mostly within the required limits from 52 to 78. Besides, the obtained estimates $\hat{\sigma}_n^2$ are slightly smaller than the true value in most cases. Only the 5-th and, in some sense, the 13-th (blue and green components) and 11-th (blue component) test images are problematic for accurate estimation of noise variance. The reason is that after downsampling these images become very textural.

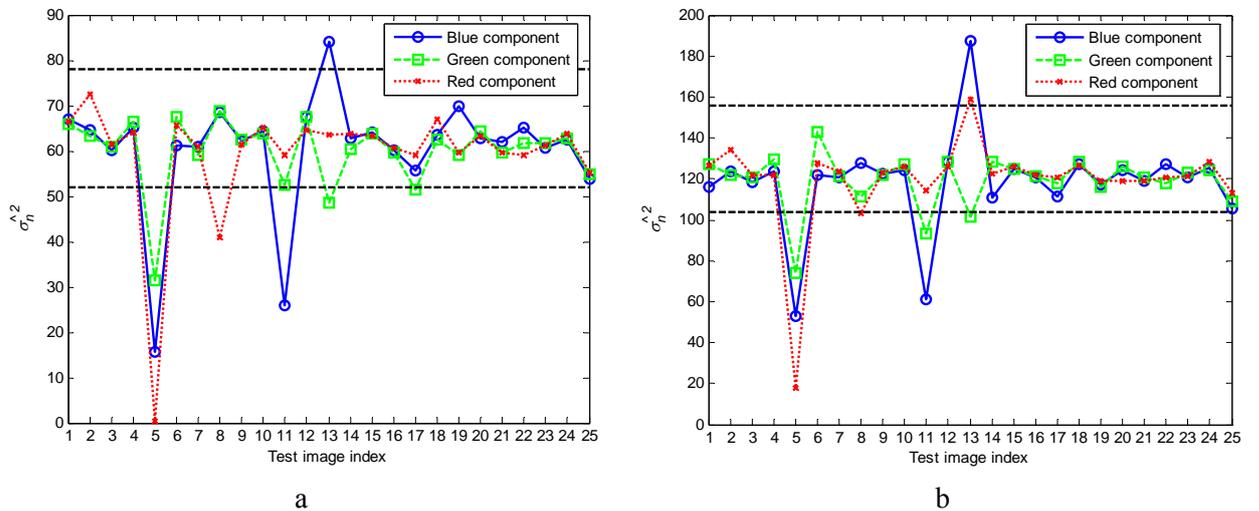


Fig. 10. Noise variance estimates obtained by the method [32, 33] for red, green and blue components for the test image set corrupted by spatially uncorrelated additive noise with $\sigma_n^2=65$ (a) and $\sigma_n^2=130$ (b)

The same method has been also tested for $\sigma_n^2=130$. The obtained data are presented in Fig. 10.b. For most cases, the obtained estimates satisfy the requirement to their accuracy (to be within the limits from 104 to 156). The exceptions are the 5-th test image (all components), the 11-th and 13-th test images (blue and green components). The reasons have been explained earlier.

Summarizing the testing results presented above, we can state the following. Although the considered methods have some differences in their main properties and theory put into their basis, they all mostly satisfy the requirements to the provided accuracy of blind estimation. However, the problem of noise variance estimation in highly textural images as the test images 5, 11, and 13 remains.

4.2. Variance estimation for spatially correlated noise

Consider the more complicated case of spatially correlated noise. Similarly to previous subsection, let us start from the method [21]. Fig. 11 represents the plots for red, green, and blue components of the test images for noise variance 65. Let the block size be 5x5 pixels. Blocks are non-overlapping.

Several main conclusions can be drawn from analysis of the plots in Fig. 11:

- 1) again for most test images the estimates $\hat{\sigma}_n^2$ for different components of the same color image are quite close; the only exception is again the 20-th test image; the reason is the clipping effect discussed above;
- 2) in most cases, the estimates $\hat{\sigma}_n^2$ are smaller or only slightly larger than the true value of variance, the only exception is the 13-th, very textural test image and, partly, the 14-th test image which is also textural;
- 3) for quite many cases, the estimates $\hat{\sigma}_n^2$ are within the required limits from 52 till 78, although quite many estimates are out of these limits, commonly smaller than the lower limit.

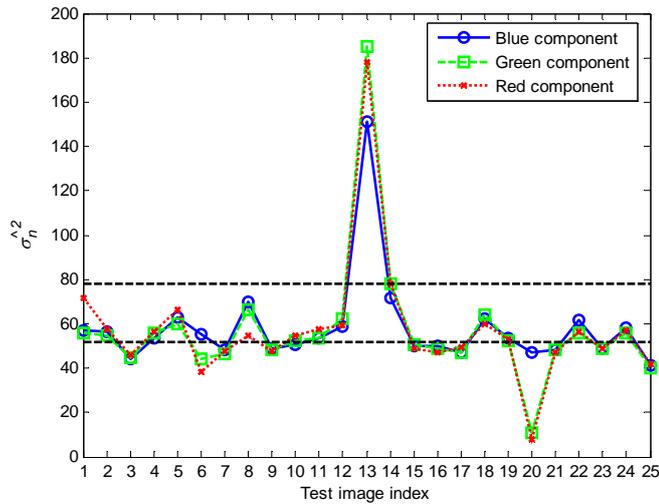


Fig. 11. Noise variance estimates obtained by the method [21] with nonoverlapping 5x5 pixels block size for red, green and blue components for the test image set corrupted by spatially correlated additive noise with $\sigma_n^2=65$

Thus, it is desirable to improve the performance of the method [21] for spatially correlated noise. Following the recommendation given in the paper [19], let us use a larger size blocks (7x7). The obtained results are presented in Fig. 12. As can be seen from comparison of the plots in Figures 11 and 12, due to using the block size 7x7 instead of 5x5 the accuracy has, in general, improved. In particular, the errors $|\hat{\sigma}_n^2 - \sigma_n^2|$ has decreased for all components of the test images 3, 4, 6, 7, 9, 10, 11, 15, 16, 17, 19, 21, 22, 23, 24, 25 and some color components of other test images. The reason for this improvement deals with the following property of normal local estimates of noise variance. The maximum of their histogram is located for $\sigma_{\max}^2 < \sigma_n^2$. But if the block size increases, the difference $\sigma_n^2 - \sigma_{\max}^2$ reduces as follows from the theory of variance estimation for data samples of limited size. This phenomenon can be also observed from analysis of the histograms in Fig. 3 in the neighborhoods of their maximums.

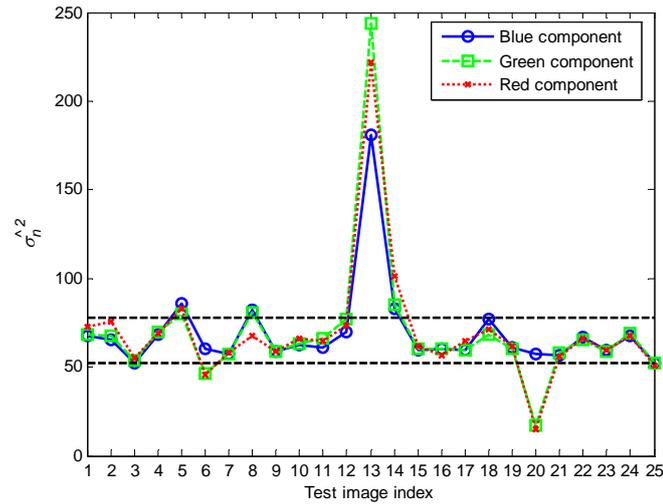


Fig. 12. Noise variance estimates obtained by the method [21] with nonoverlapping 7x7 pixels block size for red, green and blue components for the test image set corrupted by spatially correlated additive noise with $\sigma_n^2=65$

Note that improvement of accuracy due to using the 7x7 blocks has taken place for images that are not too textural. At the same time, the accuracy for textural test images like the 5-th, 13-th, and 14-th has worsened.

In aggregate, for spatially correlated noise there exists difference what block size to apply: 5x5 or 7x7. The presented results clearly show that 7x7 is a better choice. The same tendencies have been observed for other values of noise variance, in particular, 130. Because of this, below in Fig. 13 only the results for 7x7 block size are given. Mostly the obtained estimates are within the required limits. The obvious exceptions are the test images 13 and 14 (which are textural) and 20 (due to clipping effect). The estimates are also not accurate enough for some components of the textural images 1 and 8.

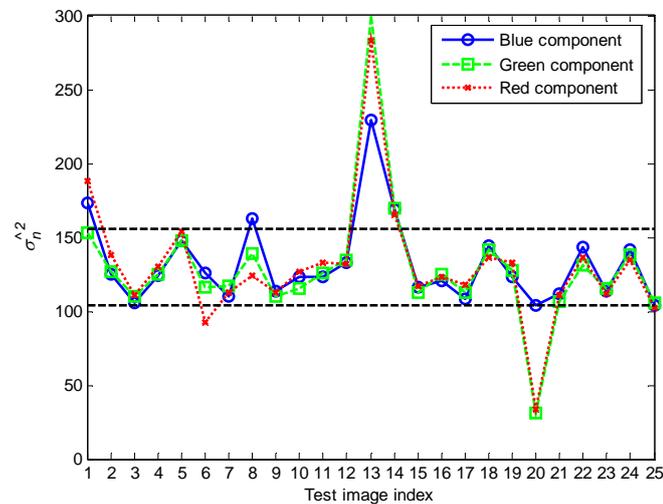


Fig. 13. Noise variance estimates obtained by the method [21] with nonoverlapping 7x7 pixels block size for red, green and blue components for the test image set corrupted by spatially correlated additive noise with $\sigma_n^2=130$

Let us now present the results obtained for the method [18] if noise is spatially correlated and its variance is equal to 65. The plots are given in Fig. 14. Their analysis clearly shows the drawbacks of this method – it totally fails for the case of spatially correlated noise with rather wide main lobe of 2D autocorrelation function. The same level of estimates is observed for the methods [20] and [31] (not presented in plots). This shows the problems of the methods that relate to the second group.

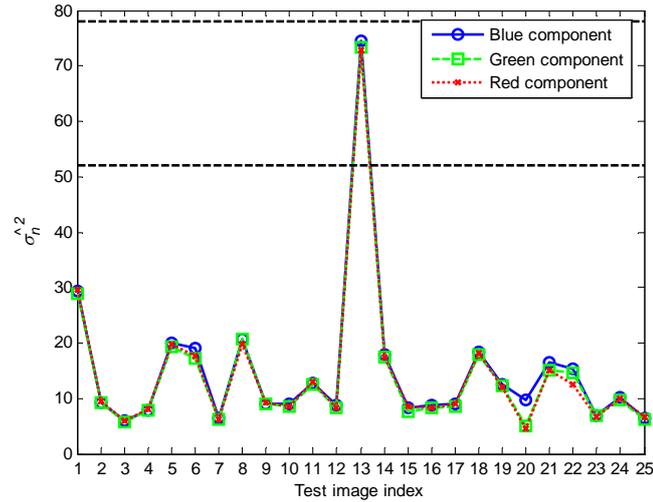


Fig. 14. Noise variance estimates obtained by the method [18] for red, green and blue components for the test image set corrupted by spatially correlated additive noise with $\sigma_n^2=65$

Being applied without any modification to blind estimation of spatially correlated noise variance, the original method [33] based on maximum likelihood estimation also produces considerably biased estimates $\hat{\sigma}_n^2 < \sigma_n^2$. However, an alternative modification used in this paper that carries out resampling (downsampling) an original image performs well enough. The data obtained for the proposed modification for $\sigma_n^2 = 65$ are presented in Fig. 15.

The estimates are mostly within the required limits. However, there are test images for which the obtained estimates $\hat{\sigma}_n^2$ are obviously out of them. These are the 5-th and 13-th test images for which the modified method fails. The reason is that these images are textural. More in detail, the test image 13 is highly textural both on base scale and after resampling. At the base scale, the test image 5 contains homogeneous fragments but of very small area. After resampling, linear size of these fragments decrease below scanning window size and they are not recognized as homogeneous. Thus, after resampling the test image 5 becomes highly textural too.

The outlying estimate of noise variance is also obtained for the blue component of the 11-th test image. There are two reasons behind this. The first reason is that clipping effects in dark homogeneous areas that occupy rather large space of entire image are observed. The second reason is that after resampling the image also becomes highly textural.

Very similar results have been obtained for $\sigma_n^2 = 130$. Therefore, some problems with highly textural images are typical for both original and modified methods based on ML estimation.

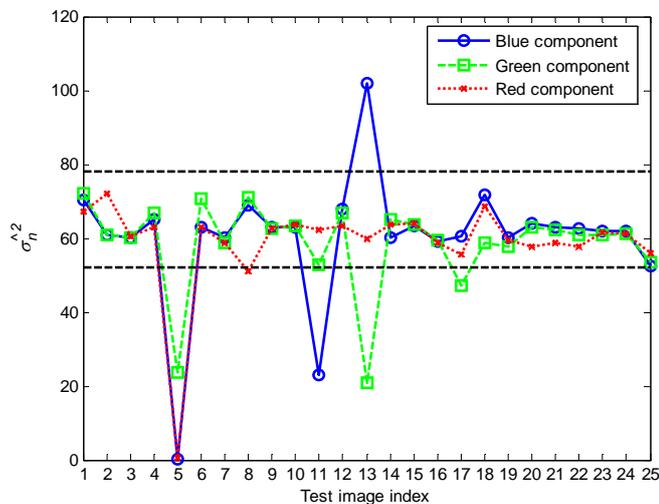


Fig. 15. Noise variance estimates obtained by the modified method [33] for red, green and blue components for the test image set corrupted by spatially correlated additive noise with $\sigma_n^2=65$

4.3. Problems of variance estimation

The analysis carried out for three groups of methods for blind evaluation of additive noise variance applied to TID2008 images has demonstrated the following:

- 1) for most images which are natural (not artificial), all three groups (under certain conditions) are able to provide the required accuracy;
- 2) the most complex situation is with highly textural images especially if they are corrupted by spatially correlated noise with rather small variance; then it is difficult to find regions or features by exploiting which it is possible to separate image information content and noise and, hence, to estimate noise variance with proper accuracy;
- 3) analysis of plots of noise variance estimates for different images shows that their distribution is not Gaussian (for a given variance of noise); outlying estimates can be observed for images which are, e.g., highly textural.

This shows that it is desirable to design special indicators of such image/noise situations as well as to continue elaboration of methods applicable to such situations. There are several ideas that can occur fruitful in this sense; in particular, recently a new entropy-like measure [35] able to characterize image content has been introduced, a coordinate of robust kurtosis-like parameter histogram maximum [36] seems to contain information on noise and image properties, the parameter p used in the method [21] for characterizing the percentage of blocks belonging to image homogeneous regions might be useful; however, these and other parameters need additional studies to give final practical recommendations.

These are general conclusions. But there are also particular conclusions and considerations concerning aforementioned problems. The first group of methods includes quite many methods although only one of them is studied in details above. It is worth saying that several methods that belong to the first group have been designed by the authors of this book chapter [1, 4, 14, 19, 21, 30, 37]. During several years we were steadily improving performance of our methods by the following directions. They include improving the robustness of the estimators used at the final stage [4, 21, 30], removal of abnormal estimates by pre-segmentation [19, 29], method parameter selection (optimization) [19], etc. Comparisons have been done to earlier versions as well as to other methods of the first group like the method [17]. Comparison has been also done to the methods of other groups [19], e.g., to the method [18]. Analysis has been carried out for several standard gray scale test images as Barbara, Baboon, Goldhill, Peppers, etc. Practically in all considered cases the accuracy provided by our methods was better or at the same level as for other techniques.

The latest version of our techniques is the method [19] that exploits image pre-segmentation. Let us see does it produce some improvements for the case where the method [21] has produced not accurate estimates. Consider the most “problematic” (textural) test images # 5, 8, 13, and 14 for spatially uncorrelated noise with $\sigma_n^2 = 65$. The results for the method [19] are presented in Table 1 (the estimates obtained without pre-segmentation are given in parentheses for convenience). As seen, the use of pre-segmentation leads to considerably more accurate estimation. In many cases, the estimates have become fitting the required limits. Only for the 13th test image and the red component of the 14th test image the estimates are out the limits. After pre-segmentation, only a very small part of the 13th test image is recognized as homogeneous. It is placed in the top part of the image where there is a small piece of sky, However, even this fragment is textural in blue component and subject to clipping for the green and red components. These effects explain the properties of the estimates obtained for the components of this image. Note that for the first group of methods [19, 21] we have not applied detection and removal of clipped fragments.

Thus, analysis carried out for the method [19] shows that pre-segmentation is worth applying.

Table 1. Variance estimates for the test images # 5, 8, 13, and 14, $\sigma_n^2 = 65$, i.i.d. noise

Channel	Blue	Green	Red
5	62.2 (89.8)	66.5 (96.9)	74.1 (91)
8	76.2 (97.8)	77.8 (90.3)	61.7 (84.2)
13	109.9 (185)	25.3 (253)	38.4 (223)
14	69.6 (90.9)	69.6 (105)	82.1 (98.3)

One advantage of the methods of the first group is that they are quite fast. Less than 1s is needed for applying them to 512x512 image using middle level modern computers to get the final estimate even if full overlapping of blocks is used.

Let us consider the method recently proposed by Barducci et al [6]. In general, it cannot be referred to any of the three groups mentioned above. This method is based on a novel principle: analysis of noise in bit-planes of considered images. Our preliminary analysis has shown that the original method [6] is very sensitive to clipping effects. If these effects are present for even a small percentage of pixels, the algorithms fails to perform appropriately well. The reason for this is that each bit-plane becomes non-random in clipped areas due to saturation. This violates the main idea put behind the bit-plane method [6] that less significant bit-planes are affected only by noise and are random.

Because of this, to improve the algorithm performance we had to introduce the following modification. First, we have detected clipped areas following the rule described above in subsection 4.1. Then, these areas have been removed from further consideration when calculating bit-plane randomness criterion $\delta(k)$ [6]. Thus, we avoid final variance estimation from being affected by clipping effect.

These modifications have considerably decreased the algorithm sensitivity to clipping effects. However, the algorithm accuracy has anyway remained not well enough. To prove this, Fig. 16 presents the plots of noise variance estimates for the TID2008 images corrupted by spatially uncorrelated noise with variance 65. Similarly, the plots for i.i.d. additive noise with $\sigma_n^2 = 130$ are demonstrated in Fig. 17. The following conclusions can be drawn:

- 1) most estimates are considerably larger than the true value of noise variance, only few estimates are within the required limits;
- 2) especially large estimates (considerably larger than they have to be) are observed for highly textural images.

Analysis for spatially correlated noise has been carried out as well. The estimates are, in general, smaller and more estimates satisfy accuracy requirements. However, anyway the accuracy of the method [6] is worse than the accuracy of the methods of the earlier considered groups. In particular, this follows from comparison of the plots presented in Figures 18 and 12.

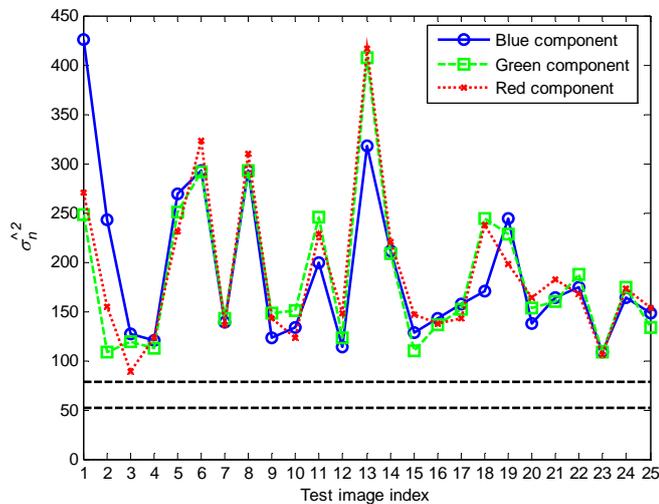


Fig. 16. Noise variance estimates obtained by the modified method [6] for red, green and blue components for the test image set corrupted by spatially uncorrelated additive noise with $\sigma_n^2=65$

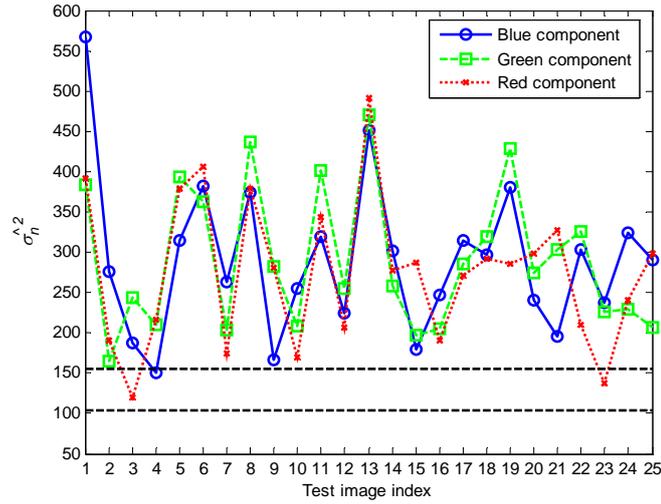


Fig. 17. Noise variance estimates obtained by the modified method [6] for red, green and blue components for the test image set corrupted by spatially uncorrelated additive noise with $\sigma_n^2=130$

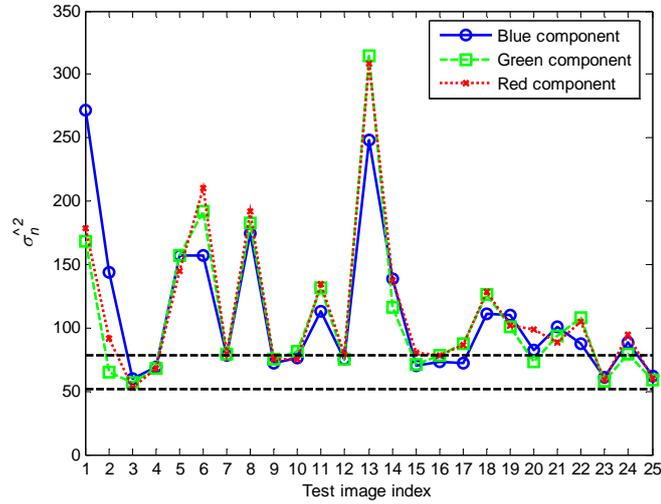


Fig. 18. Noise variance estimates obtained by the modified method [6] for red, green and blue components for the test image set corrupted by spatially correlated additive noise with $\sigma_n^2=65$

As it has been already stated and demonstrated, the methods of the second group fail if noise is spatially correlated. One way out is to “recognize” images corrupted by spatially correlated noise. This is still an open problem. If it will be solved, then it will become possible to apply, for example, the methods of the first group if noise is recognized as spatially correlated and to apply a second group method, e.g., the method [31] if noise is i.i.d. Another way out seems to be applying downsampling of an original image corrupted by spatially correlated noise as it occurred useful for the methods of the third group.

The methods of the second group are quite fast since they are based on orthogonal transforms that usually have fast algorithms and on different sorting operations that can be performed rather quickly as well.

The main advantage of the third group of methods is that they provide more accurate and stable results for highly textural images, thus widening the class of images for which reliable noise variance estimation can be obtained. Yet, abnormal biases for images 5, 13 and 11 show that further advancing of these methods is needed. One drawback of the third group of methods from practical point of view is that they are rather slow in comparison to the most methods of the first and second groups. The reason for this is extensive computations for underlying texture parameters estimation. This means that the methods of the third group should be used only for images with complex structure. In other cases, faster methods of the first or second group are worth applying. To implement this idea in a fast and blind way, some methods for image complexity estimation are to be developed.

Let us characterize accuracy and robustness of the considered methods by the parameter N_{out} – the number of estimates that are out the required limits for 75 images (three color components of 25 test images for fixed variance of noise). These data are collected in Table 2. Their analysis confirms the main conclusions given above. In particular, it shows that better accuracy (smaller N_{out}) is provided if noise variance is larger. The methods of the third group produce the best accuracy. The methods of the first and the second group are characterized by approximately the same accuracy for spatially uncorrelated noise with large variance, but if the noise is spatially correlated the methods of the second group fail.

Table 2. The values N_{out} for different estimation methods and noise properties

Method	Method parameters	Noise variance	i.i.d. or spatially correlated	N_{out}
Of the first group from the paper [21]	Block size - 7x7, non-overlapping blocks	65	i.i.d.	25
		130	i.i.d.	20
		65	spat. correlated	14
Of the second group from the paper [18]		65	i.i.d.	41
		130	i.i.d.	25
		65	spat. correlated	72
Of the second group from the paper [31]		65	i.i.d.	2
		130	i.i.d.	2
		65	spat. correlated	75
Of the third group from the paper [33]	With image down-sampling by two times	65	i.i.d.	7
		130	i.i.d.	7
		65	spat. correlated	8
by Barducci et al [6]	Modified to avoid clipping effects	65	i.i.d.	75
		130	i.i.d.	72
		65	spat. correlated	49

If noise is spatially correlated, one should keep in mind that blind estimation of noise variance or standard deviation can be only a preliminary step to further analysis of noise properties. The next step could be blind estimation of noise spatial spectrum or 2D autocorrelation function. One approach how to do this is described in the paper [38]. Obviously, a priori information on noise standard deviation allows determining image homogeneous regions for which quite accurate estimates of noise spatial spectrum can be obtained. Then, these estimates can be “aggregated” (processed jointly in some robust manner) to produce more accurate estimates of spatial spectrum to be further used for image filtering, restoration, edge detection, etc.

Conclusions and future work

In this chapter, we have tested several methods for blind evaluation of additive noise variance that belong to different groups (according to operation principles put behind them). Testing has been performed for TID2008 that contains 25 color images with different structure and properties. Moreover, we have considered two variances of noise as well as both spatially uncorrelated and correlated noise.

The testing has demonstrated that although considerable efforts have been spent in recent years on design and performance improvement of these methods, the work is far from completeness. First, only the methods of the first group can be sought as those ones providing appropriate accuracy in about 80% of the considered situations under condition that 7x7 blocks are used. The methods of the second and the third groups provide good accuracy if noise is spatially uncorrelated. However, if noise is spatially correlated this should be known in advance to undertake the corresponding decisions and modifications. Thus, a particular future task is design of methods for blind determination is an image corrupted by i.i.d. or by spatially correlated noise.

The analysis carried out has confirmed one more time that the most complex and unfavorable practical situation is when an analyzed image is textural, noise is spatially correlated and has rather small variance. Then it is the most difficult situation in the sense of providing the required accuracy of noise variance estimation in blind (automatic) manner. Note that in such situations it is difficult to do this in interactive manner as well since image homogeneous regions can be hardly selected. Saying “hardly”, we mean that even an experienced expert cannot be absolutely confident that selected “homogeneous” fragments are really homogeneous.

The study has also shown that clipping effects have to be taken into account. Although it is understood that this should be so, designers of practical realizations (codes) of the proposed methods often “forget” that clipping should be taken into consideration. Moreover, it can be a good option to indicate image regions where clipping takes place. This can be useful for further image processing since clipping can influence different operations of image processing as edge detection, classification, filtering, restoration in a negative way.

Although we have carried out component-wise analysis of noise statistics of color images, one interesting observation is the following. If an estimate of noise variance is larger (smaller) than the true value in one component, the estimates in other components are, most likely, larger (smaller) than the true value as well. In other words, the estimates in different components of color images are highly correlated. This deals with two facts. The first known fact is that components of color images are correlated [39]. The other fact is that although all methods of blind evaluation of noise variance attempt to separate image content and noise, image content anyway influences estimation of noise parameters.

Finally, we would like to mention the following. All methods considered above are based on initial assumption that noise is additive with spatially invariant (constant) variance. Although in many fundamental books (see, e.g., [40]) it is stated that this is a typical model of noise in color and other types of images, this is only a simplified model. Due to a set of operations carried out with input data in imaging systems, noise statistical properties can be quite complicated [2, 5, 41]. Then, the use of blind estimation methods adapted to pure additive noise to analysis of images corrupted by real life noise might lead to unexpected and hardly tractable results. In other words, we would like to say – be convinced that noise is pure additive before applying the methods considered above to real life images.

References

1. B. Vozel, S. Abramov, K. Chehdi, V. Lukin, N. Ponomarenko, M. Uss, Blind methods for noise evaluation in multi-component images, book chapter in “Multivariate image processing”, ISTE Ltd, France, 2009 (in press).
2. A. Foi, Pointwise Shape-Adaptive DCT Image Filtering and Signal-Dependent Noise Estimation: Thesis for the degree of Doctor of Technology, Tampere University of Technology, Tampere, Finland, Dec. 2007.
3. J.-S. Lee, K. Hoppel and S.A. Mango, Unsupervised Estimation of Speckle Noise in Radar Images, *International Journal of Imaging Systems and Technology*, Vol. 4, 1992, pp. 298-305.
4. Zelensky A.A., Totsky A.V., Lukin V.V. et al, Airborne multichannel remote sensing data processing techniques and software, *Proceedings of the Second International Airborne Remote Sensing Conference and Exhibition ERIM, San Francisco, CA, USA, June 1996, Vol. III*, pp. 151-159.
5. A. Barducci, D. Guzzi, P. Marcoionni, and I. Pippi, “CHRIS-Proba performance evaluation: signal-to-noise ratio, instrument efficiency and data quality from acquisitions over San Rossore (Italy) test site”, *Proceedings of the 3-rd ESA CHRIS/Proba Workshop, Italy*, 11 p., March 2005.
6. Barducci, A.; Guzzi, D.; Marcoionni, P.; Pippi, I. Assessing Noise Amplitude in Remotely Sensed Images Using Bit-Plane and Scatterplot Approaches; *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 45, No 8, Aug. 2007, pp. 2665-2675.
7. Zlokolica V., Pizurica A., Philips W. Noise estimation for video processing based on spatio-temporal gradients, *IEEE Signal Processing Letters*, Vol. 13, No 6, 2006, pp. 337 – 340.
8. *Hyperspectral Data Exploitation: Theory and Applications*, Edited by Chein-I Chang, Wiley-Interscience, 2007.
9. Lukin V.V., *Methods and Algorithms for Pre-processing and Classification of Multichannel Radar Remote Sensing Images*, *Proceedings of International Conference "Avances en Ciencias de la Computacion"*, Mexico, Oct. 2003, pp. 28-40.
10. N. Renard, S. Bourennane, J. Blanc-Talon, Denoising and dimensionality reduction using multilinear tools for hyperspectral images, *IEEE Geoscience and Remote Sensing Letters*, Vol. 5, No 2, 2008, pp. 138-142.
11. K. Egiazarian, J. Astola, N. Ponomarenko, V. Lukin, F. Battisti, Metrics Performance Comparison for Color Image Database, CD-ROM *Proceedings of the Second International Workshop on Video Processing and Quality Metrics*, Scottsdale, USA, 2009, 6 p.
12. E. Vansteenkiste, D. Van der Weken, W. Philips, E. Kerre, Perceived image quality measurement of state-of-the-art noise reduction schemes, *Lecture Notes on Computer Science*, Springer, Vol. 4179, 2006, pp. 114-126.
13. M.-P. Carton-Vandecandelaere, B. Vozel, L. Klaine, K. Chehdi, Application to Multispectral Images of a Blind Identification System for Blur, Additive, Multiplicative and Impulse Noises, *Proceedings of EUSIPCO, Toulouse (France)*, Vol. III, pp. 283-286, 2002.
14. B. Vozel, K. Chehdi, L. Klaine, V.V. Lukin, S.K. Abramov, Noise identification and estimation of its statistical parameters by using unsupervised variational classification, *Proceedings of ICASSP*, 2006, Toulouse, France, Vol. II, pp. 841-844.

15. Yuxin Chen, M. Das, An Automated Technique for Image Noise Identification Using a Simple Pattern Classification Approach, Proceedings of 50th Midwest Symposium on Circuits and Systems, Montreal, Canada, 2007, pp. 819-822.
16. B. Vozel, K. Chehdi, N. Benazzouz, Multispectral Image Noise Identification, Proceedings of Atlantic Europe Conference on Remote Imaging and Spectroscopy, Preston, UK, 2006, pp. 2-8.
17. Rank K., Lendl M., Unbehauen R. Estimation of Image Noise Variance, IEE Proceedings on Vision, Image and Signal Processing, Vol. 146, No 2, pp. 80-84, 1999.
18. L. Sendur, I.W. Selesnick, Bivariate shrinkage with local variance estimation, IEEE Signal Processing Letters, Vol. 9, No 12, 2002, pp. 438-441 (see also <http://taco.polv.edu/WaveletSoftware/index.html>)
19. V. Lukin, S. Abramov, B. Vozel, K. Chehdi, J. Astola, Segmentation-based method for blind evaluation of noise variance in images, SPIE Journal on Applied Remote Sensing, Vol. 2, Aug. 2008, 15 p. (open access paper)
20. Ponomarenko N.N., Lukin V.V., Abramov S.K., Egiazarian K.O., Astola J.T. Blind evaluation of additive noise variance in textured images by nonlinear processing of block DCT coefficients, Proceedings of International Conference "Image Processing: Algorithms and Systems II", Santa Clara, CA, USA, SPIE Vol. 5014, 2003, pp. 178-189.
21. Lukin V.V., Abramov S.K., Vozel B., Chehdi K. Improved minimal inter-quantile distance method for blind estimation of noise variance in images, Proceedings of SPIE/EUROPTO Symposium on Satellite Remote Sensing, Florence, Italy, Sept 2007, SPIE Vol. 6748, 12 p.
22. Abramov S.K., Lukin V.V., Ponomarenko N.N., Egiazarian K.O., Pogrebnyak O.B., Influence of multiplicative noise variance evaluation accuracy on MM-band SLAR image filtering efficiency, Proceedings of the Fifth International Kharkov Symposium "Physics and Engineering of Millimeter and Sub-Millimeter Waves", Kharkov, Ukraine, June 2004, Vol.1, pp. 250-252.
23. Melnik V.P., Lukin V.V., Zelensky A.A., Astola J.T., Kuosmanen P., Local Activity Indicators: Analysis and Application to Hard-Switching Adaptive Filtering of Images, Optical Engineering Journal, Vol. 40, No 8, pp. 1441-1455, 2001.
24. N. Ponomarenko, M. Carli, V. Lukin, K. Egiazarian, J. Astola, F. Battisti, Color Image Database for Evaluation of Image Quality Metrics, Proceedings of International Workshop on Multimedia Signal Processing, Australia, Oct. 2008, pp. 403-408.
25. K. Egiazarian, J. Astola, N. Ponomarenko, V. Lukin, F. Battisti, M. Carli, New full-reference quality metrics based on HVS, CD-ROM Proceedings of the Second International Workshop on Video Processing and Quality Metrics, Scottsdale, USA, 2006, 4 p.
26. V. Lukin, N. Ponomarenko, K. Egiazarian, J. Astola, Adaptive DCT-based filtering of images corrupted by spatially correlated noise, Proc. SPIE Conference Image Processing: Algorithms and Systems VI, Vol. 6812, 12 p., 2008.
27. Foi A., M. Trimeche, Katkovnik V., Egiazarian K. Practical Poissonian-Gaussian Noise Modeling and Fitting for Single Image Raw Data, IEEE Transactions on Image Processing, Vol. 17, No 10, 2007, pp. 1737-1754.
28. Smolka B., Plataniotis K.N., Venetsanopoulos A.N., Nonlinear Techniques for Color Image Processing, Chapter 12 in Nonlinear Signal and Image Processing: Theory, Methods, and Applications, Electrical Engineering & Applied Signal Processing Series, ed. by K. Barner and G. Arce, CRC Press, 2003, 560 p.
29. Lukin V.V., Abramov S.K., Ponomarenko N.N., Vozel B., Chehdi K., Methods for blind evaluation of noise variance in multichannel optical and radar images, Telecommunications and Radioengineering, Vol. 65 (6), 2006, pp. 509-537.
30. V.V. Lukin, S.K. Abramov, A.A. Zelensky, J.T. Astola, Use of minimal inter-quantile distance estimation in image processing, Proceedings of SPIE Conference on Mathematics of Data/Image Pattern Recognition, Compression, and Encryption with Applications IX, SPIE Vol. 6315, San Diego, USA, 2006, 12 p.
31. N. Ponomarenko, V. Lukin, M. Zriakhov, A. Kaarna, J. Astola, An automatic approach to lossy compression of AVIRIS images, Proceedings of IGARSS, July 2007, Barcelona Spain, pp. 472-475.
32. Uss M., Lukin V., Abramov S., Vozel B., Chehdi K., Joint Estimation of Multiplicative and Impulsive Noise Parameters in Remote Sensing Images with Fractal Structure, Proceedings of ICASSP, April 2007, Honolulu, USA, Vol. 1, pp. 581-584.
33. Uss M., Lukin V., Baryshev I., Vozel B., Chehdi K., Joint Estimation of Additive and Impulsive Noise Parameters in Remote sensing Images with Fractal Structure, Proceedings of the International Conference "Modern Problems of Radioengineering, Telecommunications and Computer Science" (TCSET), Lviv-Slavsko, Ukraine, Feb.-March 2006, pp. 232-235.
34. Foi A., Practical Denoising of Clipped or Overexposed Noisy Images, CD ROM Proceedings of EUSIPCO, 2008, 5 p.
35. S.K. Abramov, V.V. Lukin, N.N. Ponomarenko, O.B. Pogrebnyak, Entropy-like measure of background content for image retrieval and sorting in large databases, Telecommunications and Radio Engineering, Vol. 68, No 8, pp. 667-675.

36. V. Lukin, D. Fevralev, S. Abramov, N. Ponomarenko, S. Krivenko, O. Pogrebnyak, K. Egiazarian, Igor Djurovič, Removal of non-stationary noise by DCT based locally adaptive filtering, Practical aspects of digital signal processing, Shakhty, Russia, 2009.
37. Lukin V.V., Abramov S.K., Vozel B., Chehdi K. A method for blind automatic evaluation of noise variance in images based on bootstrap and myriad operations, Proceedings of SPIE/EUROPTO Symp. On Satellite Remote Sensing, Bruges, Belgium, Sept 2005, SPIE Vol. 5982, pp. 299-310.
38. V.V. Lukin, N.N. Ponomarenko, S.K. Abramov, B. Vozel, K. Chehdi, J. Astola, Filtering of radar images based on blind evaluation of noise characteristics, Proceedings of Image and Signal Processing for Remote Sensing XIV, Cardiff, UK, Sept 2008, SPIE Vol. 7109, 12 p.
39. Astola J., Haavisto P. Neuvo Y. Vector Median Filters, Proceedings of IEEE, 1990, Vol. 78, pp. 678-689.
40. K.N. Plataniotis, A.N. Venetsanopoulos, Color Image Processing and Applications, NY, Springer-Verlag, 2000.
41. Paliy D. Local approximations in demosaicing and deblurring of digital sensor data: Thesis for the degree of Doctor of Technology, Tampere University of Technology, Tampere, Finland, Dec. 2007.